Exercise walk-through

2.1
Domains, inverses and composition

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

2.1 Domains, inverses and composition

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS domain

Objective

Domains, inverses and composition. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

The method

- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

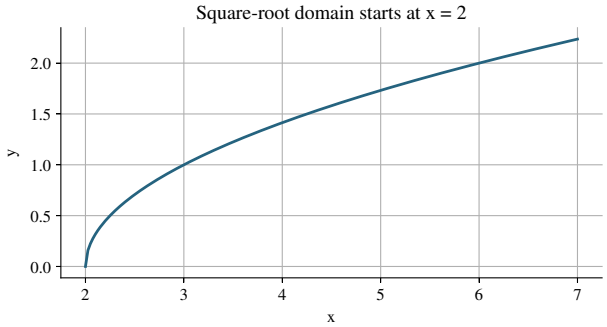
A worked case

$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

Example:

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=y^2+2. Thus f in-

verse(x)=x^2+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.



Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for x^2), so its domain must be restricted first.

Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Domain**: The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

Write from an inverse value of 6

- The function 6x plus 1 sends 6 to 37.
- A quadratic on all real numbers needs a restricted domain before it has an inverse.

Exercises 2.1·2.3

2.1 For f(x)=2x+3, find f inverse(11). Show the relation, substitution and units. [2]

2.2 For f(x)=2x+3 and g(x)=x^2, find fg(4). Show the relation, substitution and units. [2]

2.3 For f(x)=√(x-2), find f(11). Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For f(x)=2x+3, find f inverse(11). Show the relation, substitution and units. [2]

2.1 Solve 2x+3=11, giving x=4.

2.2 For f(x)=2x+3 and g(x)=x^2, find fg(4). Show the relation, substitution and units. [2]

2.2 Apply g first: g(4)=16; then f(16)=2×16+3=35.

2.3 For f(x)=√(x-2), find f(11). Show the relation, substitution and units. [2]

2.3 Substitute into the square root: √(11-2)=√9=3.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.5 Why must $f(x)=x^2$ have its domain restricted before an inverse is defined? Choose one answer.

- **A** Squaring is undefined for negative inputs, so those inputs must be removed first
- **B** Two different inputs share each output, so the reverse is not a function
- **C** The range of x^2 is empty unless the domain is restricted to $x \geq 0$ [1]

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.4 False. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

2.5 Why must $f(x)=x^2$ have its domain restricted before an inverse is defined? Choose one answer.

- **A** Squaring is undefined for negative inputs, so those inputs must be removed first
- **B** Two different inputs share each output, so the reverse is not a function
- **C** The range of x^2 is empty unless the domain is restricted to $x \geq 0$ [1]

2.5 B x^2 sends both 3 and -3 to 9, so an unrestricted inverse could not choose a single value. Restricting to $x \geq 0$ makes the function one-to-one and the inverse $\sqrt{x}$ well defined.

Exercise 3.1

3.1 For $f(x)=x^2$ on $x\geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16). Explain why the restriction matters. [4]

Exercise 3.1

3.1 For $f(x)=x^2$ on $x\geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16). Explain why the restriction matters. [4]

3.1 Composite$=(x-3)^2$ with $x\geq 3$ because $g(x)$ must lie in f's domain. Inverse $16=4$. Without restriction x^2 is not one-to-one and the inverse would not be a function on the full real domain.