

Exercise walk-through

1.5

Complex numbers and roots

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.5 Complex numbers and roots

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS complex conjugate

Objective

Complex numbers and roots. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

■ The method

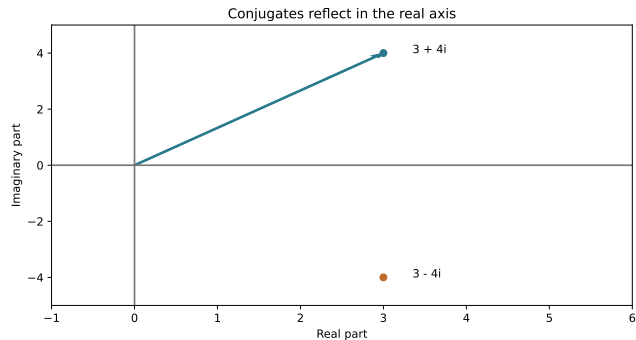
- Write $z=a+bi$ with $i^2=-1$.
 - The conjugate is $a-bi$ and z times its conjugate= a^2+b^2 .
 - The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant.
 - Conjugate roots occur for polynomials with real coefficients.
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■ A worked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

Example:

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



■ Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

Not true: For $a+bi$, its modulus is always $\sqrt{a^2+b^2}$. The modulus is $\sqrt{a^2+b^2}$: for $3+4i$ it is 5, not 7.

■ Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Complex conjugate:** The number obtained by reversing the sign of the imaginary part.
- Write $z=a+bi$ with $i^2=-1$.
- Watch for: the square root of a sum is not the sum of the square roots.

■ **Write from a modulus of 13**

- Find the modulus of 5 plus 12 i.
- The modulus is not the sum of the two parts.

Exercises 2.1·2.3

2.1 Find $|3+4i|$. Show the relation, substitution and units. [2]

2.2 Find the real part of $(3+4i)(3-4i)$. Show the relation, substitution and units. [2]

2.3 Find the real part of $1/(3+4i)$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find $|3+4i|$. Show the relation, substitution and units. [2]

2.1 Modulus= $\sqrt{(3^2+4^2)}=5$.

2.2 Find the real part of $(3+4i)(3-4i)$. Show the relation, substitution and units. [2]

2.2 Conjugate multiplication gives $3^2+4^2=25$, with imaginary parts cancelling.

2.3 Find the real part of $1/(3+4i)$. Show the relation, substitution and units. [2]

2.3 Multiply numerator and denominator by $3-4i$: $1/(3+4i)=(3-4i)/25$. Its real part is $3/25=0.12$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For $a+bi$, its modulus is always $a+b$.^[2]

2.5 Why do we multiply $1/(3+4i)$ by $(3-4i)/(3-4i)$? Choose one answer.

- **A** It removes the real part, leaving a purely imaginary denominator
- **B** It doubles the modulus of the denominator, which makes the division easier
- **C** The conjugate product $(3+4i)(3-4i)=25$ is real, clearing the complex denominator^[1]

3.1 Find all complex roots of $z^2=-8i$ in Cartesian form and verify them. ^[4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For $a+bi$, its modulus is always $a+b$. [2]

2.4 False. The modulus is $\sqrt{a^2+b^2}$: for $3+4i$ it is 5, not 7.

2.5 Why do we multiply $1/(3+4i)$ by $(3-4i)/(3-4i)$? Choose one answer.

- **A** It removes the real part, leaving a purely imaginary denominator
- **B** It doubles the modulus of the denominator, which makes the division easier
- **C** The conjugate product $(3+4i)(3-4i)=25$ is real, clearing the complex denominator [1]

2.5 C z times its conjugate is $|z|^2=a^2+b^2$, a real number. Dividing by that real value leaves a clean $a+bi$ form: $(3-4i)/25$.

3.1 Find all complex roots of $z^2=-8i$ in Cartesian form and verify them. [4]

3.1 Polar modulus 8 and argument $-\pi/2$ give roots modulus $\sqrt{8}$, arguments $-\pi/4$ and $3\pi/4$: $2-2i$ and $-2+2i$. Squaring each gives $-8i$. Both roots are needed.