

Exercise walk-through

1.4

Percentages, ratio and proportional reasoning

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.4 Percentages, ratio and proportional reasoning

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS multiplier

Objective

Percentages, ratio and proportional reasoning. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** : A factor that performs a percentage change in

one multiplication.

■ The method

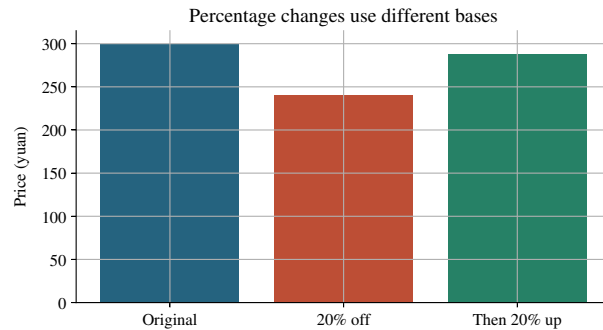
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
 - Reverse a percentage by dividing by the multiplier.
 - In a ratio, first find the total number of parts.
-

■ A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

Example:

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



■ Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

■ Applying it

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$.
- Calculate k from a known pair before using a new value.
- State what you held constant.

■ In your course

- Analysis and Approaches HL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Multiplier:** A factor that performs a percentage change in one multiplication.
 - A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
 - Watch for: A percentage uses a stated base.
-

■ **Write from a final price of 240**

- A price of 250 falls by 20 percent and then rises by 20 percent.
- Find the final price. The two changes do not cancel.

Exercises 2.1-2.3

2.1 A price of 80 rises by 15%. Find the new price. Show the relation, substitution and units. [2]

2.2 A 25% discount leaves a price of 90. Find the original price. Show the relation, substitution and units. [2]

2.3 Share 84 in the ratio 2:5. Find the larger share. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 A price of 80 rises by 15%. Find the new price. Show the relation, substitution and units. [2]

2.1 Multiply by 1.15: $80 \times 1.15 = 92$.

2.2 A 25% discount leaves a price of 90. Find the original price. Show the relation, substitution and units. [2]

2.2 The sale price is 75% of the original: $90 / 0.75 = 120$.

2.3 Share 84 in the ratio 2:5. Find the larger share. Show the relation, substitution and units. [2]

2.3 There are seven ratio parts. The larger share is $84 \times 5 / 7 = 60$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Multiplier means a factor that performs a percentage change in one multiplication. [2]

2.5 A price falls 20% then rises 20%. Why is the final price not the starting price? Choose one answer.

- **A** The 20% rise is taken of the original price, so it cancels exactly
 - **B** Percentage changes always cancel when they are equal and opposite
 - **C** The second 20% acts on the reduced price, not the original
- [1]

3.1 A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Multiplier means a factor that performs a percentage change in one multiplication. [2]

2.4 True. True: A factor that performs a percentage change in one multiplication.

2.5 A price falls 20% then rises 20%. Why is the final price not the starting price? Choose one answer.

- **A** The 20% rise is taken of the original price, so it cancels exactly
- **B** Percentage changes always cancel when they are equal and opposite
- **C** The second 20% acts on the reduced price, not the original [1]

2.5 C A 20% decrease multiplies by 0.8 and the later 20% increase multiplies that result by 1.2: $0.8 \times 1.2 = 0.96$. Each percentage uses a different base.

3.1 A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair. [4]

3.1 Factor $1.2 \times 0.8 = 0.96$, a 4% fall. Recovery factor $1/0.96$, giving 4.1667% increase. Both percentage changes use different bases.