

8.3 Sets and hash tables

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS union • intersection • difference • set • hash table
• hash function • collision • chaining • linear probing
• load factor

Objective

Build the skills to answer HL exam questions on **sets and hash tables** – set operations, hash functions, collisions, and the load factor.

You must be able to:

- state the result of **union** , **intersection** and **difference** on two **sets**
- place keys in a **hash table** using a given **hash function**
- resolve a **collision** by **chaining** or **linear probing**
- explain how the **load factor** affects performance

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

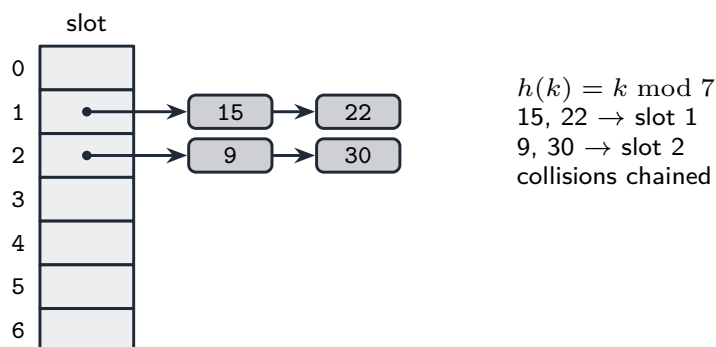
■ Key knowledge: sets

A set holds distinct values with no order. For $a = \{1, 2, 3, 4\}$ and $b = \{3, 4, 5\}$:

Operation	Python	Result
union	<code>a b</code>	$\{1, 2, 3, 4, 5\}$
intersection	<code>a & b</code>	$\{3, 4\}$
difference	<code>a - b</code>	$\{1, 2\}$
membership	<code>5 in a</code>	<code>False</code>

- `a.add(2)` changes nothing: 2 is already there.
- Sets and dictionaries are hash tables, so `in` is $O(1)$ on average; on a list it is $O(n)$.

■ Worked example: hash table with chaining



Seven slots, $h(k) = k \bmod 7$, keys 15, 22, 9, 30.

1. $15 \bmod 7 = 1$: slot 1.
2. $22 \bmod 7 = 1$: a collision; chaining adds 22 to slot 1's list.
3. $9 \bmod 7 = 2$ and $30 \bmod 7 = 2$: slot 2 holds $9 \rightarrow 30$.
4. To find 30: hash to slot 2, then search its short list.

■ Worked example: linear probing

Same keys, but each slot holds one key; on a collision, try the next slot, wrapping from 6 to 0.

Key	$k \bmod 7$	Slots tried	Placed in
15	1	1	1
22	1	1, 2	2
9	2	2, 3	3
30	2	2, 3, 4	4

A search for 30 follows the same path and stops at 30, or at an empty slot (not found).

■ Key knowledge: load factor

- Load factor = $\frac{\text{items stored}}{\text{slots}}$: here $\frac{4}{7} \approx 0.57$.
- A higher load factor means more collisions and longer searches; the table is enlarged and every key rehashed.

2 Practice

2.1 $P = \{2, 4, 6, 8\}$ and $Q = \{4, 8, 12\}$. State $P \cup Q$, $P \cap Q$ and $Q - P$. [3]

2.2 Using $h(k) = k \bmod 10$ and 10 slots with chaining, state the slot of each key: 47, 12, 57, 30, 92. Name the slots holding chains. [3]

2.3 Insert 47, 12, 57, 30, 92 into the same table using linear probing instead. State the slot of each key. [3]

2.4 State the load factor of the table in 2.3. [1]

3 Exam-style questions

3.1 Which data structure guarantees that no value is stored twice? [1]

- ☐ **A** a list
☐ **B** a stack
☐ **C** a set
☐ **D** a queue

3.2 What is a collision in a hash table? [1]

- ☐ **A** two keys hash to the same slot
☐ **B** the table is full
☐ **C** a key is not found
☐ **D** two values are equal

3.3 Why is a hash table usually faster to search than a sorted array? [1]

- ☐ **A** it keeps its keys in order
☐ **B** the hash function gives the slot directly, $O(1)$ on average
☐ **C** it never has collisions
☐ **D** it uses less memory

3.4 A school library stores books by 5-digit ID in a hash table of 11 slots, numbered 0 to 10, with $h(k) = k \bmod 11$ and linear probing.

(a) Insert the IDs 10023, 10034, 10045 and 10001 in that order, showing the hash value and slot of each. [4]

(b) State the slots examined when searching for 10045. [1]

(c) Explain why a search for a missing ID can stop at an empty slot. [2]

(d) The library expects 2000 books. Explain why a table of 11 slots is unsuitable, and suggest a size. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $P \cup Q = \{2, 4, 6, 8, 12\}$
- $P \cap Q = \{4, 8\}$
- $Q - P = \{12\}$

2.2

- $47 \rightarrow 7, 12 \rightarrow 2, 57 \rightarrow 7, 30 \rightarrow 0, 92 \rightarrow 2$
- chains in slots **2** and **7**

2.3

- $47 \rightarrow 7, 12 \rightarrow 2, 30 \rightarrow 0$
- 57: 7 taken, slot **8**
- 92: 2 taken, slot **3**

2.4

- $\frac{5}{10} = \mathbf{0.5}$

3.1 C

- a set holds distinct values only

3.2 A

- two keys, one slot

3.3 B

- no search through the keys is needed

3.4

(a)

- $10023 \bmod 11 = 2$: slot **2**
- $10034 \bmod 11 = 2$: taken, slot **3**
- $10045 \bmod 11 = 2$: 2 and 3 taken, slot **4**
- $10001 \bmod 11 = 2$: 2, 3, 4 taken, slot **5**

(b)

- slots **2, 3, 4**

(c)

- if the ID had been inserted, it would have gone into the first empty slot on its probe path
- so reaching an empty slot means it is not in the table

(d)

- 11 slots cannot hold 2000 keys with probing; with chaining, chains would be about 180 long, so searches are slow
- a size giving a load factor well below 1, such as 4001 slots (a prime about twice the number of books)