Reaction energies, activity and decay balances

GRE undergraduate preparation · local supported objectives

Nuclear energy release, activity and decay chains

- Find the **Q value** Q 值 of a reaction from atomic masses.
- Balance charge and nucleon number.
- Use **activity** 活度, the decay constant and half-life.

Nuclear energy release, activity and decay chains (2)

- Follow decay chains to **secular equilibrium 长期平衡**.
- Read the binding-energy curve.

Q values

- $Q > 0$: kinetic energy is released. $Q < 0$: energy must be supplied.
- Tables give atomic masses, which include electrons. They cancel for alpha and beta-minus decay. For beta-plus decay subtract $2m_e c^2 = 1.022$ MeV.

$$Q = (m_{\text{initial}} - m_{\text{final}})\, c^2, \qquad 1 \text{ u} \to 931.5 \text{ MeV}$$

Balance first

- $^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \to {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{p}$: charge $7 + 2 = 8 + 1$, nucleons $14 + 4 = 17 + 1$.
- Alpha decay lowers Z by 2 and A by 4; beta-minus raises Z by 1 and leaves A unchanged.

Activity

- Activity is measured in becquerel: 1 Bq $= 1$ decay per second.
- Two samples with equal activities can hold very different numbers of atoms, if their half-lives differ.

$$A = \lambda N, \qquad \lambda = \frac{\ln 2}{T_{1/2}}, \qquad A(t) = A_0 e^{-\lambda t}$$

Decay chains

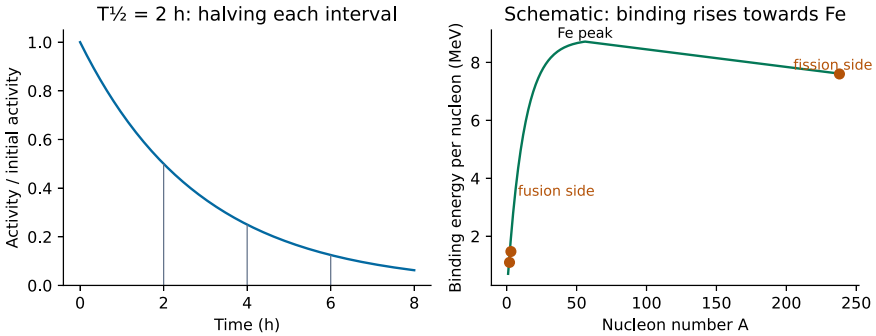
- If the parent lives much longer than the daughter, the daughter's activity rises to equal the parent's: **secular equilibrium**.
- Radium-226 (1600 years) and radon-222 (3.8 days) reach it within about a month.

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

The binding curve

- Fusing light nuclei or splitting heavy ones both move toward the peak and release energy.
- Fission of $A \approx 240$: about $(8.5 - 7.6) \times 240 \approx 200$ MeV.

Figure: The binding curve



- Each half-life halves the activity; energy is released by moving towards the iron peak, by fusion from the left or fission from the right
- Read the axes and units first.

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Worked example: Q values

- $^{238}\mathrm{U} \to {}^{234}\mathrm{Th} + \alpha$:
  $(238.0508 - 234.0436 - 4.0026) \times 931.5 = 0.0046 \times 931.5 = 4.3$ MeV.
- Momentum is shared equally, so the light alpha takes about $\frac{234}{238}$ of it, 4.2 MeV.
- $\mathrm{D} + \mathrm{T} \to {}^{4}\mathrm{He} + \mathrm{n}$:
  $(2.0141 + 3.0160 - 4.0026 - 1.0087) \times 931.5 = 0.0188 \times 931.5 = 17.5$ MeV.

Worked example: activity

- $\lambda = 0.693/7200 = 9.6 \times 10^{-5}$ s^{-1}.
- $A = 9.6 \times 10^{-5} \times 6.0 \times 10^{12} = 5.8 \times 10^{8}$ Bq.
- After 6.0 h, three half-lives, the activity is $\frac{1}{8}$ of this, 7.2×10^{7} Bq.

Common errors

- Computing Q before checking the balance.
- Leaving out the two electron masses in beta-plus decay.
- Using the half-life in place of λ in $A = \lambda N$.

Common errors (2)

- Assuming equal activities mean equal numbers of atoms.
- Thinking only fission releases energy, or that fusing iron does.

Key terms

- **fission**: The splitting of a heavy nucleus into two lighter ones.
- **mass defect**: The difference between the mass of the separate parts and the mass of the bound system.
- **Q value**: The rest energy turned into kinetic energy in a reaction, $(m_{\text{initial}} - m_{\text{final}})c^2$.
- **activity**: The decay rate λN of a radioactive sample, in becquerel.

Check yourself

- What particle is missing in ${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + \ ?$
- A sample's activity falls to $\frac{1}{16}$ in 20 days. What is its half-life?
- Why does fusing two iron nuclei not release energy?