

Reaction energies, activity and decay balances
GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

One atomic mass unit is 931.5 MeV, so a reaction energy is a small difference between large masses. Compute reaction energy from complete atomic or nuclear mass bookkeeping Calculate activity from half-life and a simple decay-chain balance Compare fission and fusion binding changes with charge and nucleon conservation

Bookkeep the masses

Reasoning

The reaction energy is $Q=(m_{\text{initial}}-m_{\text{final}})c^2$; $Q>0$ releases kinetic energy. Track every species, and keep the mass convention consistent: tabulated atomic masses include electrons. For β^- decay and for α decay with a neutral helium product, atomic electron counts balance directly; β^+ or electron capture needs explicit electron-mass terms. Use $1\text{ u}=931.5\text{ MeV}/c^2$.

Compute Q and check balance

Reasoning

Alpha decay of ^{238}U with atomic masses $238.0508\text{ u} \rightarrow 234.0436\text{ u} + 4.0026\text{ u}$ gives $\Delta m = 0.0046\text{ u}$ and $Q \approx 4.28\text{ MeV}$. Fusion $\text{D} + \text{T}$ with $2.0141 + 3.0161 - 4.0026 - 1.0087 = 0.0189\text{ u}$ gives $Q \approx 17.6\text{ MeV}$. A reaction also requires charge and nucleon-number balance: $^{14}\text{N} + \alpha \rightarrow ^{17}\text{O} + \text{p}$ balances $7 + 2 = 9$ and $14 + 4 = 18$.

Convert half-life to activity

Reasoning

Activity is $A = \lambda N$ with decay constant $\lambda = \ln 2 / T^{1/2}$; one becquerel is one decay per second. For $T^{1/2} = 2$ h and $N = 6 \times 10^{12}$, $A = 6 \times 10^{12} \times \ln 2 / 7200 \approx 5.8 \times 10^8$ Bq. After three half-lives both N and A fall by $1/8$. Equal activities do not imply equal atom counts: shorter half-life at fixed N gives larger A .

Compare binding changes

Reasoning

In a chain $A \rightarrow B \rightarrow C$ with a long-lived parent ($\lambda_A \ll \lambda_B$), B accumulates until its activity approaches the parent activity (secular equilibrium); write the balance $dN_B/dt = \lambda_A N_A - \lambda_B N_B$. The binding-energy-per-nucleon curve peaks near iron, so heavy fission (about 7.6 to 8.5 MeV per nucleon over roughly 240 nucleons, of order 200 MeV) and light fusion both release energy; compare total binding, not nucleon counts.

Worked reasoning 1

Reasoning

Activity is $A = \lambda N$, with $\lambda = \ln 2 / T_{1/2}$. For $N = 6.0 \times 10^{12}$ and half-life 2 hours, convert time before calculating.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

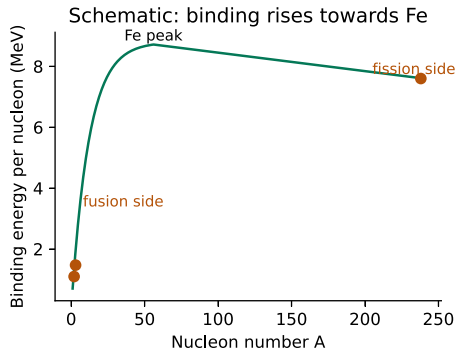
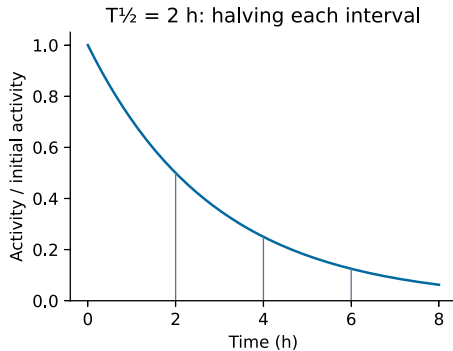
$$A = (\ln 2)N/T_{1/2} = (\ln 2)(6.0 \times 10^{12})/(7200 \text{ s}) = 5.78 \times 10^8 \text{ Bq}.$$

Worked reasoning 3

Reasoning

A decay chain requires both production and loss terms: $\dot{N}_B = \lambda_A N_A - \lambda_B N_B$. Equal activities do not imply equal populations. Atomic-mass beta-plus Q values require subtracting $2m_e c^2$.

Read the original diagram



- Original GRE teaching diagram: Reaction energies, activity and decay balances
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A parent has initial population N_0 and daughter population zero. Both decay constants equal λ . Derive daughter population and the time of its maximum. **[5]**

Independent transfer · reasoning

Reasoning

Parent population is $N_A = N_0 e^{-\lambda t}$. Daughter equation $\dot{N}_B + \lambda N_B = \lambda N_0 e^{-\lambda t}$ has integrating factor $e^{\lambda t}$, giving $N_B = \lambda N_0 t e^{-\lambda t}$. Derivative is $\lambda N_0 e^{-\lambda t} (1 - \lambda t)$; maximum occurs at $t = 1/\lambda$, with $N_B = N_0/e$. Award parent, equation, integrating factor/solution, maximum time and value. Direct use of an unequal-decay-constant formula would give a removable zero-over-zero limit.

Check assumptions and limits

Reasoning

Mixing atomic and nuclear masses without tracking electrons, or reporting a positive Q for a reaction that needs energy input. Balance charge and nucleon number first, then check the sign of Q .

Keep the technical distinctions

Reasoning

Q value Q : The rest-mass energy converted to kinetic energy in a reaction, $(m_{\text{initial}} - m_{\text{final}})c^2$. activity : The decay rate λN of a radioactive sample, measured in becquerel.

Check yourself

- Define Q value and give its condition of use.
- Define activity and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.