Radiation attenuation, laser gain and cavity modes

GRE undergraduate preparation · local supported objectives

Radiation in matter and laser cavities

- Find the range of charged particles.
- Use exponential **attenuation** 衰减 and the **half-value layer** 半值层 for photons.
- Explain **stimulated emission** and the need for **population inversion**.
- Find the frequencies and spacing of **cavity modes** 腔模.

Charged particles: a definite range

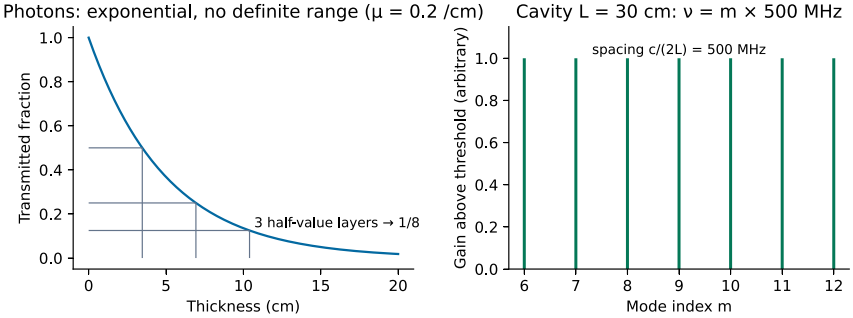
- With a constant energy loss per distance, range $= E_0/(dE/dx)$. A 5.0 MeV particle losing 2.0 MeV/cm stops after 2.5 cm.
- Real energy loss grows as the particle slows, so most of the energy is deposited near the end of the track.
- Electrons scatter strongly and radiate, so their range is less sharp.

Photons: exponential attenuation

- With $\mu = 0.20\ \text{cm}^{-1}$, the half-value layer is 3.5 cm, and 10 cm transmits $e^{-2} = 0.135$.
- Three half-value layers transmit $\frac{1}{8}$.
- A broad beam transmits more than this, because scattered photons still reach the detector (build-up).

$$I = I_0\, e^{-\mu x}, \qquad x_{1/2} = \frac{\ln 2}{\mu}$$

Figure: Photons: exponential attenuation



- Identify the axes, units, directions and the stated model.

Stimulated emission and inversion

- Absorption competes with it, so net gain needs $N_2/g_2 > N_1/g_1$, where g is each level's degeneracy.
- In thermal equilibrium the upper level always has fewer atoms, so heating a two-level system never gives an inversion.
- Lasers pump a third level that decays quickly into a long-lived upper laser level.

Cavity modes

- A 30 cm cavity has modes 500 MHz apart.
- A mode lases only if its gain is larger than its round-trip losses.

$$\nu_m = \frac{mc}{2L}, \qquad \Delta\nu = \frac{c}{2L}$$

Worked example: a shield

- Half-value layer: $0.693/0.20 = 3.5$ cm.
- After 10 cm: $e^{-2.0} = 0.135$ of the beam remains.
- For a $\frac{1}{16}$ count rate with a detector of efficiency 0.25, the shield need only transmit $\frac{1}{4}$: thickness $\ln 4/\mu = 6.9$ cm.

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Worked example: a laser

- Two levels 2.0 eV apart at 300 K ($k_B T = 0.0259$ eV): $N_2/N_1 = e^{-77} \approx 10^{-34}$. Without pumping, almost no atoms are excited.
- A 25 cm cavity: $\Delta\nu = 3.0 \times 10^8 / 0.50 = 600$ MHz.
- If the gain is above the losses over 1.5 GHz, about $1.5/0.60 \approx 2$ or 3 modes can lase.

Common errors

- Giving photons a definite range, or charged particles an exponential one.
- Using μ in place of $\ln 2/\mu$ for the half-value layer.
- Leaving detector efficiency out of a count rate.

Common errors (2)

- Thinking heating can produce a **population inversion**.
- Using c/L for the mode spacing; the round trip is $2L$.

Key terms

- **attenuation**: The reduction in intensity of a beam as it passes through matter.
- **half-value layer**: The thickness of material that halves a narrow beam, $\ln 2/\mu$.
- **population inversion**: A state with more atoms per sublevel in the upper laser level than the lower.
- **stimulated emission**: Emission caused by an incoming photon, giving a copy of that photon.

Check yourself

- A beam passes through four half-value layers. What fraction remains?
- What is the mode spacing of a 1.5 m laser cavity?
- Why can a two-level system in thermal equilibrium not lase?