

Radiation attenuation, laser gain and cavity modes

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A photon beam has no definite range, yet a heavy charged particle does; shielding calculations fail if the two are swapped. Distinguish charged-particle stopping and range from exponential photon attenuation Compute attenuated intensity and detected counts with stated coefficients Explain stimulated emission, population inversion and optical cavity resonance conditions

Choose the interaction model

Reasoning

Heavy charged particles lose energy continuously through many small collisions, so they have an approximate range: under a constant-loss model a particle with initial energy E_0 and loss rate dE/dx stops after $E_0/(dE/dx)$. Electrons straggle more and radiate. Photons instead interact in single events, so a narrow beam attenuates exponentially as $I=I_0 e^{(-\mu x)}$ with no definite maximum depth; the half-value layer is $\ln 2/\mu$.

Attenuate and count

Reasoning

Apply the exponential only to the stated narrow-beam geometry: $\mu=0.2$ /cm gives a half-value layer of 3.47 cm and $I/I_0=e^{-2}\approx 0.135$ after 10 cm. Multiply by detector efficiency for recorded counts: with $\mu x=\ln 4$ and $\varepsilon=0.25$ the recorded fraction is $0.25\times 1/4=1/16$. Build-up from scattered photons makes broad-beam shielding transmit more than the narrow-beam exponential; state the geometry.

Invert the population

Reasoning

Stimulated emission produces a photon matching the stimulating photon in frequency, direction and phase, giving coherent amplification. It competes with absorption; net gain needs population inversion $N_2/g_2 > N_1/g_1$ ($N_2 > N_1$ for equal degeneracies). Positive-temperature equilibrium has $N_2/g_2 < N_1/g_1$ by the Boltzmann factor, so a two-level system in equilibrium cannot lase continuously; practical lasers pump a third level or a metastable state.

Resonate in the cavity

Reasoning

A linear cavity of length L supports standing modes with $L=m\lambda/2$, i.e. frequencies $\nu_m=mc/(2L)$ with spacing $c/(2L)$: for $L=30$ cm the spacing is 500 MHz. Gain must exceed the round-trip losses for oscillation. Interferometers such as Michelson use the same coherence: fringe counts track optical path changes, as in the gas-cell measurement.

Worked reasoning 1

Reasoning

A narrow beam has attenuation $I = I_0 e^{-\mu x}$ and half-value layer $x_{1/2} = \ln 2 / \mu$.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$x_{1/2} = (\ln 2)/(0.20 \text{ cm}^{-1}) = 3.47 \text{ cm}.$$

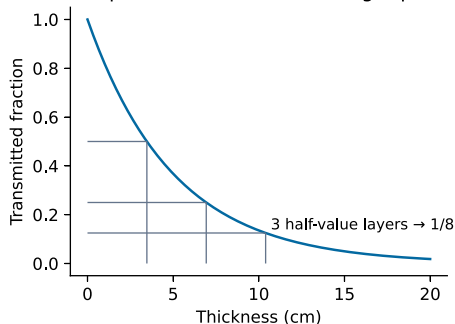
Worked reasoning 3

Reasoning

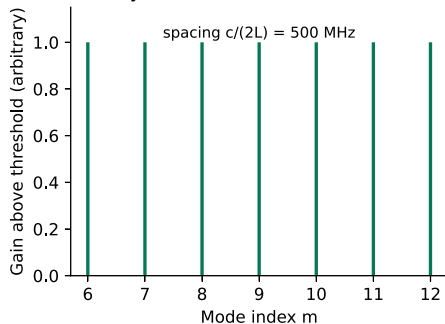
Three half-value layers transmit $1/8$. This model excludes scattered build-up into the detector. For a linear vacuum laser cavity, adjacent longitudinal modes are separated by $c/(2L)$. Net stimulated gain requires $N_2/g_2 > N_1/g_1$; the simpler $N_2 > N_1$ assumes equal degeneracies.

Read the original diagram

Photons: exponential, no definite range ($\mu = 0.2$ /cm)



Cavity $L = 30$ cm: $\nu = m \times 500$ MHz



- Original GRE teaching diagram: Radiation attenuation, laser gain and cavity modes
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A cavity of length L contains a gain medium over its entire length with intensity gain coefficient g , distributed intensity loss coefficient α , and mirror intensity reflectivities R_1 and R_2 . Derive the threshold gain. State the intensity/amplitude convention. **[4]**

Independent transfer · reasoning

Reasoning

One round trip multiplies intensity by $R_1 R_2 e^{2(g-\alpha)L}$. Threshold requires that factor equal one, so $g_{th} = \alpha + \ln[1/(R_1 R_2)]/(2L)$. Above threshold unsaturated gain must exceed this loss; steady lasing involves saturation. Award round trip, threshold condition, logarithmic result and convention/saturation. An amplitude coefficient convention would introduce a factor of two and must be declared.

Check assumptions and limits

Reasoning

Assigning photons a definite range or charged particles a single exponential law, and forgetting that equilibrium two-level populations cannot invert. Check which interaction model the beam species requires.

Keep the technical distinctions

Reasoning

half-value layer : The material thickness that halves an exponentially attenuated beam, equal to $\ln 2 / \mu$. population inversion : A non-equilibrium state with more population in the upper laser level than the lower, enabling net stimulated emission.

Check yourself

- Define half-value layer and give its condition of use.
- Define population inversion and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.