Measurement loading, amplifier gain and detector response

GRE undergraduate preparation · local supported objectives

Laboratory instruments, meters and detectors

- Correct a meter reading for **loading** 负载效应.
- Find the gains of ideal op-amp circuits.
- Use the **cutoff frequency** 截止频率 of an RC filter.

Laboratory instruments, meters and detectors (2)

- Report detector efficiency and energy resolution.
- Correct a count rate for **dead time** 死时间.

Meter loading

- The reading is always low. A meter with $R_m = 10R_{\text{th}}$ reads 90.9% of the true value.
- A meter's **input impedance 输入阻抗** should be much larger than the source resistance. A $10\times$ oscilloscope probe raises it, but also divides the signal by 10.
- An ammeter loads a circuit the other way: its resistance must be small.

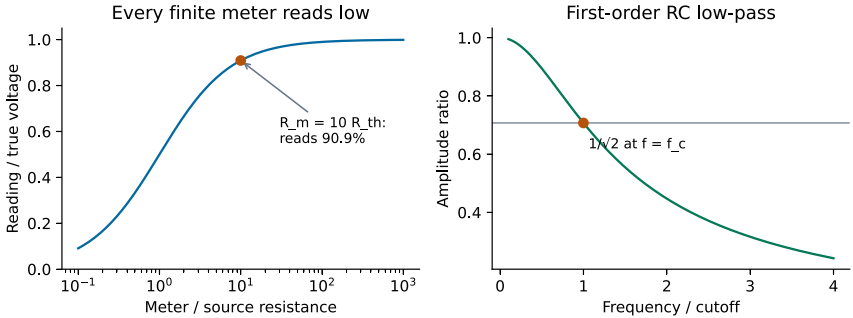
$$V_{\text{read}} = V_{\text{th}} \frac{R_m}{R_m + R_{\text{th}}}$$

Op-amps and filters

- Non-inverting gain: $1 + R_2/R_1$. Inverting gain: $-R_2/R_1$.
- An RC low-pass filter has $f_c = \dfrac{1}{2\pi RC}$.
- At f_c the amplitude ratio is $1/\sqrt{2} = 0.707$ (half the power), not $\frac{1}{2}$, and the phase is $-45°$.

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\sqrt{1 + (f/f_c)^2}}, \qquad \phi = -\arctan\frac{f}{f_c}$$

Figure: Op-amps and filters



- Identify the axes, units, directions and the stated model.

Detectors

- Efficiency = particles detected ÷ particles reaching the detector.
- Energy resolution = full width at half maximum ÷ energy. A 13.2 keV width at 662 keV is 2.0%.
- These are different properties: a detector can count well and still blur energies.

Dead time

- The correction matters when $m\tau$ is not small.
- After each recorded event, a non-paralysable detector cannot record another for a time τ.

$$m = \frac{n}{1 + n\tau}, \qquad n = \frac{m}{1 - m\tau}$$

Calibration and errors

- A linear calibration fixes an offset and a scale. It does not remove noise or nonlinearity.
- Averaging N readings reduces random error by $\sqrt{N}$, but leaves a systematic bias unchanged.

Worked example: a loaded source

- A 10 V source with 100 kΩ internal resistance, read by a 1.0 MΩ meter: $10 \times 1.0/1.1 = 9.09$ V.
- A 12 V source with 200 kΩ, the same meter: $12 \times 1.0/1.2 = 10.0$ V, a 17% error.
- A non-inverting amplifier with $R_1 = 1.0$ kΩ and $R_2 = 9.0$ kΩ has gain 10.

Worked example: a filter and a counter

- $f_c = 1/(2\pi \times 1600 \times 1.0 \times 10^{-7}) = 995$ Hz, about 1.0 kHz.
- At 3.0 kHz: ratio $1/\sqrt{1+9} = 0.32$, phase $-\arctan 3 = -72°$.
- A counter with $\tau = 20$ μs records 8000 per second: $n = 8000/(1 - 0.16) = 9524$ per second.

Common errors

- Taking the meter reading as the source voltage.
- Using $\frac{1}{2}$ for the amplitude at the **cutoff frequency**.
- Mixing up the inverting and non-inverting gains.

Common errors (2)

- Using $m(1 + m\tau)$ instead of $m/(1 - m\tau)$ for **dead time**.
- Expecting averaging to remove a systematic error.

Key terms

- **loading**: The change a measuring instrument makes to the circuit it measures.
- **input impedance**: The effective load that a measuring instrument presents to the circuit under test.
- **cutoff frequency**: The frequency at which a filter passes $1/\sqrt{2}$ of the input amplitude.
- **dead time**: The time after one recorded event during which a detector cannot record another.

Check yourself

- A meter with $R_m = 4R_{\text{th}}$ reads a 5.0 V source. What does it show?
- What is the phase shift of an RC low-pass filter at its **cutoff frequency**?
- A detector with $\tau = 10\ \mu$s has a true rate of 10^4 per second. What rate does it record?