Simultaneity, length measurement and velocity transformation

GRE undergraduate preparation · local supported objectives

Lorentz transformations, simultaneity and the interval

- Apply the **Lorentz transformation 洛伦兹变换** to the separation between two events.
- Show that **simultaneity 同时性** depends on the frame.
- Measure a moving length correctly.

Lorentz transformations, simultaneity and the interval (2)

- Classify separations with the **spacetime interval** 时空间隔.
- Add velocities relativistically.

Transforming separations

- Always transform Δx and Δt together; the time result depends on the distance too.
- Two events simultaneous in S ($\Delta t = 0$) but Δx apart have $\Delta t' = -\gamma v \Delta x / c^2$ in S': the event further along $+x$ happens first.

$$\Delta x' = \gamma \left(\Delta x - v \Delta t \right), \qquad \Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right)$$

Measuring a moving rod

- If that condition holds, $L = L_0/\gamma$, where L_0 is the **proper length** 固有长度, measured in the rod's rest frame.
- At $0.6c$, $\gamma = 1.25$, so a 2.0 m rod is 1.6 m long in the lab.
- Reading the two ends at different times gives a number that is not a length.

The invariant interval

- $s^2 > 0$, timelike: one event can cause the other, the order is the same in every frame, and the **proper time** 固有时 between them is s/c.
- $s^2 < 0$, **spacelike** 类空: no signal can connect them, and different frames can disagree about their order.
- $s^2 = 0$, lightlike: only a light signal connects them.

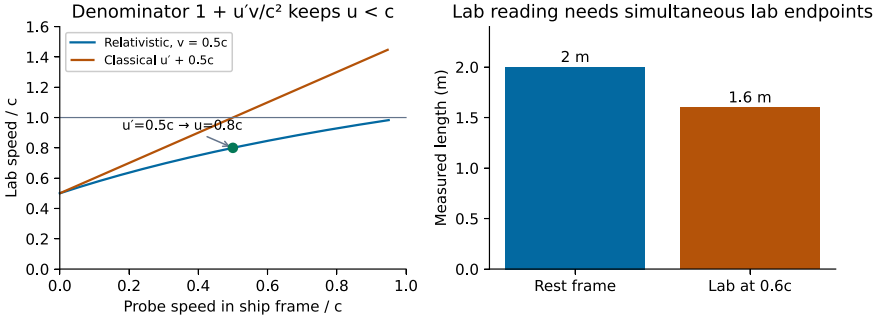
$$s^2 = c^2\Delta t^2 - \Delta x^2$$

Velocity addition

- A probe launched at $0.5c$ from a ship moving at $0.5c$ moves at $1.0c/1.25 = 0.8c$.
- With $u' = c$, $u = c$: light has the same speed in every frame.
- At low speeds the denominator is close to 1 and the result is $u' + v$.

$$u = \frac{u' + v}{1 + u'v/c^2}$$

Figure: Velocity addition



- Identify the axes, units, directions and the stated model.

Worked example: two lightning strikes

- $c\Delta t' = -\gamma(v/c)\Delta x = -1.25 \times 0.6 \times 900 = -675$ m, so $\Delta t' = -675/(3.0 \times 10^8) = -2.25$ μs.
- The passenger sees the forward strike 2.25 μs before the rear one. Both observers are right in their own frames.
- $s^2 = 0 - 900^2 < 0$: spacelike, so no frame can make one strike cause the other.

Worked example: classifying a pair

- $s^2 = 600^2 - 900^2 < 0$: spacelike.
- A frame with $v = c^2\Delta t/\Delta x = \frac{2}{3}c$ sees them simultaneous, $\sqrt{900^2 - 600^2} = 671$ m apart.
- Change to $\Delta x = 600$ m, $c\Delta t = 1000$ m: timelike, with $s = 800$ m and **proper time** $800/(3.0 \times 10^8) = 2.67\ \mu$s.

Common errors

- Transforming Δt without the $v\Delta x/c^2$ term.
- Measuring a moving rod's ends at different times.
- Contracting the **proper length** instead of the moving one, or contracting in the wrong direction.

Common errors (2)

- Adding velocities as $u' + v$ near c.
- Thinking a spacelike pair can be causally connected.

Key terms

- **Lorentz transformation**: The relativistic rule that converts event coordinates between inertial frames.
- **simultaneity**: Two events happening at the same time in a given frame.
- **spacetime interval**: $s^2 = c^2\Delta t^2 - \Delta x^2$, the same in every inertial frame; its sign classifies a separation.
- **proper length**: The length of an object measured in its own rest frame.

Check yourself

- Two events have $\Delta x = 300$ m and $c\Delta t = 500$ m. Is the pair timelike or spacelike?
- A rod has **proper length** 5.0 m. How long is it at $0.8c$?
- What is $0.8c$ added to $0.8c$?