Thermal spectra and one-electron scaling

GRE undergraduate preparation · local supported objectives

Blackbody spectra and one-electron atoms

- Use **blackbody** 黑体 radiation: Planck's law, Wien's law and the Stefan–Boltzmann law.
- Find the power radiated by a grey body with **emissivity** 发射率 ε.
- Scale Bohr-model radii, speeds and energies with n and Z.
- Calculate line energies, the **series limit** 谱线系限 and ionisation energies.

Planck's law

- Wien's law: $\lambda_{\max} T = 2.898 \times 10^{-3}$ m · K. Doubling T halves the peak wavelength.
- Stefan–Boltzmann law: total flux from the surface $= \sigma T^4$, with $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.
- The peak of the per-frequency curve B_ν is not at $c/\lambda_{\max}$; use Wien's law only with B_λ.

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Grey bodies

- Every temperature must be in kelvin; Celsius values make the answer wrong by orders of magnitude.
- A good absorber is a good emitter: the same ε appears in both terms.

$$P = \varepsilon \sigma A \left(T^4 - T_{\text{env}}^4 \right)$$

Bohr scaling

- Radius and energy carry different powers of Z: He^+ is half the size of hydrogen and four times as tightly bound.
- The Bohr model gets the energies right but not the angular momentum: the real 1s state has $l = 0$, not $L = \hbar$.

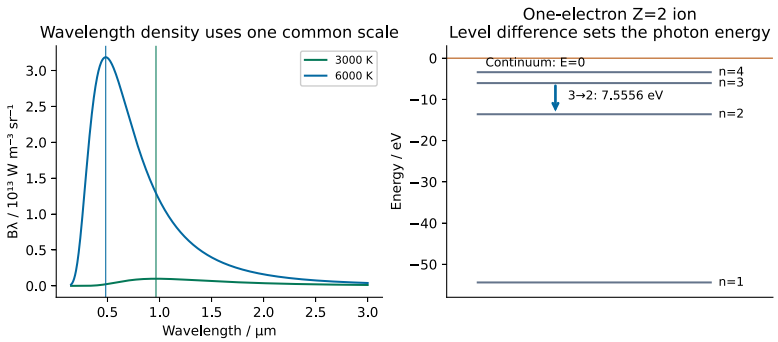
$$r_n = \frac{a_0 n^2}{Z}, \qquad v_n = \frac{Z\alpha c}{n}, \qquad E_n = -\frac{13.6\,Z^2}{n^2}\ \text{eV}$$

Lines, limits and ionisation

- The **series limit** is the photon with $n_i \to \infty$: for Balmer in hydrogen, $13.6/4 = 3.40$ eV, 365 nm.
- Ionising from level n needs $13.6\,Z^2/n^2$, which is not the same as any transition energy.

$$E_\gamma = 13.6\,Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\ \text{eV}, \qquad \lambda = \frac{1240\ \text{eV nm}}{E_\gamma}$$

Figure: Lines, limits and ionisation



- Doubling the temperature halves the peak wavelength; for a $Z = 2$ ion every level is $Z^2 = 4$ times the hydrogen value
- Read the axes and units first.

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Worked example: a hot filament

- Peak: $\lambda_{\max} = 2.898 \times 10^{-3} / 3000 = 966$ nm, in the infrared.
- Power: $0.5 \times 5.67 \times 10^{-8} \times 2.0 \times 10^{-4} \times 3000^{4} = 459$ W.
- A photon at the peak carries $1240 / 966 = 1.28$ eV.

Worked example: the helium ion

- He^{+} $3 \to 2$: $E_\gamma = 54.4\left(\frac{1}{4} - \frac{1}{9}\right) = 54.4 \times \frac{5}{36} = 7.56$ eV, so $\lambda = 1240/7.56 = 164$ nm.
- Ionising He^{+} from $n = 3$ needs $54.4/9 = 6.04$ eV.
- Its $n = 2$ orbit has radius $a_0 \times 4/2 = 2a_0$, half of hydrogen's $4a_0$.

Common errors

- Using Celsius in T^4.
- Forgetting the surroundings in the net radiated power.
- Using Wien's wavelength peak to find the frequency peak.

Common errors (2)

- Giving radius and energy the same power of Z.
- Confusing the ionisation energy from level n with a transition energy.

Key terms

- **blackbody**: An ideal surface that absorbs all radiation and emits a spectrum set only by its temperature.
- **emissivity**: The ratio of a surface's thermal radiation to that of a blackbody at the same temperature.
- **spectral radiance**: Radiated power per unit area, per unit solid angle and per unit wavelength (or frequency).
- **series limit**: The shortest-wavelength line of a series, from an initial level at $n \to \infty$.

Check yourself

- A star's spectrum peaks at 290 nm. What is its surface temperature?
- By what factor does the radiated power rise when the temperature of a blackbody doubles?
- What is the ground-state energy of Li^{2+} ($Z = 3$)?