

Thermal transport, calorimetry and expansion

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

Two wall layers in series carry the same steady heat rate, but their temperature drops need not be equal. Combine conductive thermal resistances and distinguish heat rate from heat flux Balance sensible and latent heat with stated isolation and heat-capacity conditions Calculate free thermal expansion and constrained thermal stress with their limits

Add thermal resistances

Reasoning

For one-dimensional steady conduction through a uniform slab, Fourier's law gives heat flux $q_x = -\kappa \, dT/dx$. With constant conductivity κ , area A , length L and hot-to-cold temperature difference ΔT , heat rate is $\dot{Q} = \kappa A \Delta T / L$. Define thermal resistance $R_{th} = L / (\kappa A)$, measured in K/W; temperature drop is $\dot{Q} R_{th}$. Series layers without internal sources carry the same $\dot{Q}$ and their resistances add. Parallel paths at the same endpoint temperatures have heat rates that add, so inverse resistances add. Heat flux is rate per area, W/m^2 , and need not match between layers of different area. Interface contact resistance and heat leakage must be included if specified.

Balance sensible and latent energy

Reasoning

Heat capacity $C = dQ/dT$ depends on the thermodynamic path and is not the same as specific heat c per mass. For a material interval without a phase transition, $Q = \int mc(T)dT$, or $mc\Delta T$ when c is constant. In an isolated calorimeter, sum the energy changes of all objects, including the container when relevant, and set the sum to zero. At a phase transition at its equilibrium temperature, added energy can change phase fraction rather than temperature: $Q = m\ell$ uses latent heat ℓ in J/kg. First supply the sensible heat to reach the transition, then budget latent heat. Do not let a simple weighted-temperature average predict an impossible temperature when melting or freezing is part of the process.

Check expansion constraints

Reasoning

For a freely expanding rod and small temperature change, $\Delta L \approx \alpha L_0 \Delta T$ with linear expansion coefficient α . For an isotropic solid with small strain, each dimension acquires factor $1 + \alpha \Delta T$, so $\Delta A/A \approx 2\alpha \Delta T$ and $\Delta V/V \approx 3\alpha \Delta T$. A hole expands with the surrounding material as if its missing region had expanded too. These relations assume a nearly constant coefficient and no mechanical constraint; anisotropic crystals need directional coefficients. If an elastic rod is prevented from changing length, its mechanical strain cancels thermal strain. With tensile stress positive, $\sigma \approx -Y\alpha \Delta T$ during heating, where Y is Young's modulus. This small-strain estimate requires elastic response without yielding or buckling.

Choose the transport model

Reasoning

Conduction transfers energy through a temperature gradient, convection transports it with moving matter, and thermal radiation can cross a vacuum. For a surface at T facing large surroundings at T_{env} , a simple grey-body model gives net radiative rate $\varepsilon \sigma_{\text{SB}} A(T^4 - T_{\text{env}}^4)$. Use absolute kelvin in these fourth powers, rather than Celsius values. A lumped body whose internal temperature remains nearly uniform can obey $C \, dT/dt = -hA(T - T_{\text{env}})$ under Newton cooling with constant h . Then the temperature excess decays with $\tau = C/(hA)$, rather than the entire Celsius or kelvin temperature decaying toward zero. This approximation requires sufficiently small internal gradients; a conduction-limited body needs a spatial temperature model.

Worked reasoning 1

Reasoning

Two slabs in series carry the same steady heat rate . Let resistances be 0.08 and 0.24 K/W. Boundary temperatures are 60 and 20 degrees Celsius.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$\dot{Q} = \Delta T / (R_1 + R_2) = (40 \text{ K}) / (0.32 \text{ K/W}) = 125 \text{ W}.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

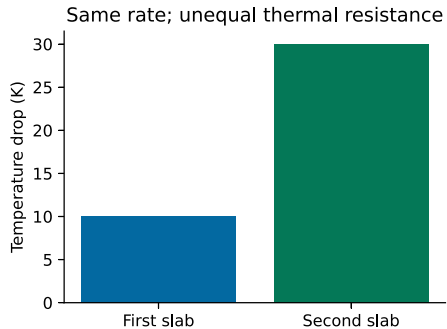
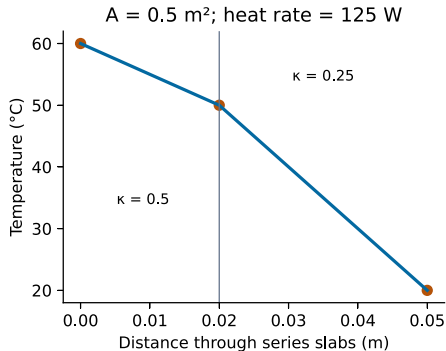
$$\Delta T_1 = \dot{Q}R_1 = (125 \text{ W})(0.08 \text{ K/W}) = 10 \text{ K}.$$

Worked reasoning 5

Reasoning

The interface is 50 degrees Celsius when slab 1 touches the hot side. Heat rate has watt units; flux divides it by area.

Read the original diagram



- Original GRE teaching diagram: Thermal transport, calorimetry and expansion
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Mix 0.20 kg of water at 40 degrees Celsius with 0.050 kg ice at zero degrees Celsius in an insulated container of negligible heat capacity. Use $c = 4200 \text{ J/(kg K)}$ and latent heat 334000 J/kg . Determine final temperature and whether ice remains. **[5]**

Independent transfer · reasoning

Reasoning

Cooling water to zero releases $mc\Delta T = (0.20)(4200)(40) = 33600$ J. Melting all ice needs $m\ell = (0.050)(334000) = 16700$ J, so all ice melts. Remaining energy heats 0.25 kg water: $T_f = (33600 - 16700)/(0.25 \times 4200) = 16.10$ degrees Celsius. Award cooling budget, melt budget, phase decision, final heat capacity and temperature. If cooling energy were insufficient, final temperature would be zero with some ice left.

Check assumptions and limits

Reasoning

Keep W separate from W/m^2 , include the calorimeter or latent energy when needed, and do not use free-expansion length together with constrained-stress assumptions.

Keep the technical distinctions

Reasoning

thermal resistance : Temperature difference per steady heat-transfer rate, measured in K/W. latent heat : Energy transferred during a phase change at its transition temperature; specific latent heat is per mass.

Check yourself

- Define thermal resistance and give its condition of use.
- Define latent heat and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.