Diffraction envelopes and missing interference orders

GRE undergraduate preparation · local supported objectives

Diffraction envelopes and missing interference orders

- Use the single-slit intensity, its **envelope 包络线** and its zeros.
- Combine slit width and slit spacing in the two-slit pattern.
- Predict **missing orders 缺级**.
- Use the **Rayleigh criterion 瑞利判据** for a circular **aperture 孔径**.

The single-slit envelope

- At $\theta = 0$ the limit is 1: the bright centre.
- Zeros at $a \sin\theta = m\lambda$, $m = \pm 1, \pm 2, \ldots$ At $\beta = \pi/2$, the intensity is $4/\pi^2 = 0.405$.
- The **far field** needs $L \gg a^2/\lambda$, or a lens that focuses the pattern.

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2, \qquad \beta = \frac{\pi a \sin\theta}{\lambda}$$

Widths on a screen

- For small angles, the first zeros are at $y = \pm L\lambda/a$, so the central peak is $2L\lambda/a$ wide.
- For large angles, use $\theta = \sin^{-1}(m\lambda/a)$ and $y = L\tan\theta$.
- A narrower slit gives a wider pattern.

Two slits: envelope times fringes

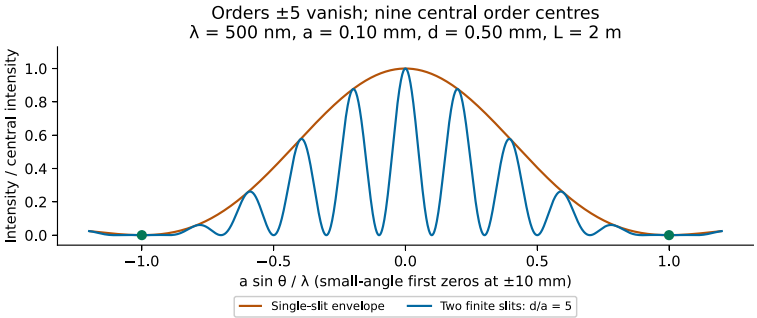
- Fringes at $d\sin\theta = n\lambda$, spaced $\lambda L/d$ on a screen.
- A larger d makes the fringes closer; a larger a makes the envelope narrower.

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha, \qquad \alpha = \frac{\pi d \sin\theta}{\lambda}$$

Missing orders

- If $d/a = r$ is a whole number, orders $\pm r, \pm 2r, \ldots$ vanish.
- The central envelope then holds $2r - 1$ fringes.

Figure: Missing orders



- Identify the axes, units, directions and the stated model.

Circular apertures and resolution

- **Rayleigh criterion**: two point sources are just resolved when the centre of one pattern falls on the first dark ring of the other.
- Bigger telescopes resolve finer detail; shorter wavelengths do too.

$$\sin\theta \approx 1.22\,\frac{\lambda}{D}$$

Worked example: two slits with missing orders

- Envelope zeros: $y = \pm L\lambda/a = \pm 10$ mm. Fringe spacing: $\lambda L/d = 2.0$ mm.
- $d/a = 5$, so the fifth orders sit at ± 10 mm, exactly on the envelope zeros: missing.
- Nine fringes, $n = -4$ to $+4$, fill the central envelope.

Worked example: seeing two headlights

- $\theta = 1.22 \times (5.5 \times 10^{-7})/(3.0 \times 10^{-3}) = 2.2 \times 10^{-4}$ rad.
- Headlights 1.5 m apart are resolved out to $1.5/(2.2 \times 10^{-4}) = 6.7$ km.
- In practice the eye does a little worse, because of imperfections and the spacing of its receptors.

Common errors

- Counting $m = 0$ as a single-slit zero.
- Mixing up the roles of a (envelope) and d (fringes).
- Expecting a fringe where the envelope is zero.

Common errors (2)

- Forgetting the factor 1.22 for a circular **aperture**.
- Using small-angle formulas at large angles.

Key terms

- **envelope**: The single-slit pattern that sets the brightness of the two-slit fringes.
- **missing orders**: Interference maxima that vanish because they fall on single-slit zeros.
- **far field**: The region far enough from an **aperture** that the pattern keeps its shape as it spreads.
- **Rayleigh criterion**: Two sources are just resolved when one's central maximum lies on the other's first minimum.

Check yourself

- What is the single-slit intensity at $\beta = \pi/2$, as a fraction of the centre?
- Two slits have $d/a = 3$. Which orders are missing, and how many fringes are in the central envelope?
- A slit is made narrower. What happens to the envelope?