

Diffraction envelopes and missing interference orders

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

Two equal slits can have the right path difference for constructive interference and still deliver no intensity at that angle. Derive and evaluate the far-field single-slit intensity using amplitude superposition. Separate double-slit interference spacing from finite-aperture envelope width. Identify missing orders and state the limits of far-field and small-angle formulas.

Add aperture amplitudes

Reasoning

For a uniformly illuminated slit of width a , far-field contributions across its opening arrive with a phase gradient $k \sin\theta$. Add their complex amplitudes before squaring: the normalised integral over x from $-a/2$ to $a/2$ is $\sin\beta/\beta$, where $\beta = \pi a \sin\theta/\lambda$. The intensity ratio is $(\sin\beta/\beta)^2$. At $\theta=0$ take the limit $\sin\beta/\beta \rightarrow 1$, rather than calling the centre undefined or dark. Zeros occur at $a \sin\theta = m\lambda$ with nonzero integer m . The central peak lies between the first zeros and is twice as wide as one adjacent zero-to-zero interval in $\sin\theta$. Side-peak maxima are not exactly halfway between their zeros.

Convert angles to the screen

Reasoning

At small angles on a distant screen L away, $y \approx L\theta$ and $\sin\theta \approx \theta$ give first zeros $y \approx \pm L\lambda/a$. Thus the central width is $2L\lambda/a$. Use metres consistently: a millimetre slit and a nanometre wavelength differ by six powers of ten. For a wider angle, use $\theta = \text{asin}(m\lambda/a)$ and $y = L \tan\theta$; the small-angle y expression then becomes inaccurate. Far-field conditions need nearly parallel rays from different parts of the aperture, for example L much greater than a^2/λ , or the equivalent focal-plane arrangement. Near-field Fresnel patterns cannot be assigned this intensity law blindly.

Multiply interference and envelope

Reasoning

For two coherent identical uniformly illuminated slits of width a and centre separation d , the normalised pattern is $(\sin\beta/\beta)^2 \cos^2\alpha$, with $\alpha=\pi d \sin\theta/\lambda$ and central intensity as the normalisation. The $\cos^2$ factor produces interference orders $d \sin\theta=n\lambda$; the single-slit factor gives the broader envelope zeros. Small-angle neighbouring interference spacing is $\lambda L/d$. Increasing d narrows fringe spacing; increasing a narrows the envelope. These are separate changes. The equal-height narrow-slit interference formula alone does not predict the diminishing brightness or missing orders of finite apertures. Incoherent sources do not maintain the same phase-dependent cross term.

Cancel coincident orders

Reasoning

A missing order occurs when $n\lambda/d = m\lambda/a$ simultaneously, giving $n = m d/a$. If d/a is an integer r , orders $\pm r$, $\pm 2r$ and so on are cancelled by aperture zeros. Do not count an order at an envelope boundary as a visible bright fringe. In the central envelope, the nominal interference-order centres satisfy $|n| < d/a$; when r is an integer there are $2r-1$ such centres. For finite slit width, exact local maxima are shifted slightly by the changing envelope, so the interference-order locations are an approximation to observed peak centres. Identify what quantity is being requested before treating every $\cos^2$ maximum as an exact maximum of the product.

Worked reasoning 1

Reasoning

For two finite slits, separate interference

spacing from the single-slit envelope

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Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

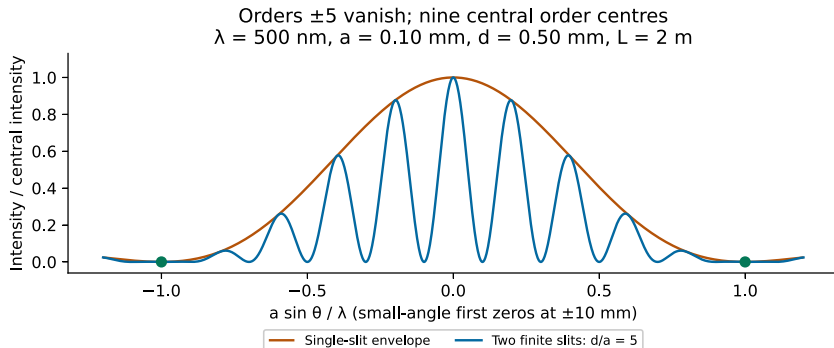
$$I/I_{\text{centre}} = (\sin \beta / \beta)^2 \cos^2 \alpha, \quad \beta = \pi a \sin \theta / \lambda, \quad \alpha = \pi d \sin \theta / \lambda.$$

Worked reasoning 3

Reasoning

If $d = 5a$, the first envelope zero $a \sin \theta = \lambda$ coincides with order $n = 5$. Nominal order centres in the central envelope are $n = -4$ through $+4$: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.

Read the original diagram



- Original GRE teaching diagram: Diffraction envelopes and missing interference orders
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A double slit has $d = 3a/2$. Which positive interference orders are missing? How many nominal interference centres lie strictly between the first envelope zeros? Explain why rounding d/a to an integer is invalid. **[4]**

Independent transfer · reasoning

Reasoning

Coincidence requires $n = m(d/a) = 3m/2$ with both m, n nonzero integers. Thus m must be even and missing orders are $n = 3, 6, 9, \dots$. Central centres obey $|n| < 3/2$, so $n = -1, 0, 1$: three. Fractional d/a describes actual geometry; rounding changes which zeros coincide. Award integer condition, missing sequence, count and explanation.

Check assumptions and limits

Reasoning

Add field amplitudes before squaring, distinguish a from d , and exclude dark boundary orders. An interference maximum alone does not guarantee a maximum of the full pattern.

Keep the technical distinctions

Reasoning

diffraction envelope : The aperture-dependent intensity factor that modulates the interference pattern. missing order : An interference order cancelled because it coincides with an aperture intensity zero.

Check yourself

- Define diffraction envelope and give its condition of use.
- Define missing order and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.