# RC and RL transients, impedance and resonance

GRE undergraduate preparation · local supported objectives

# RC and RL transients, impedance and resonance

- Solve RC and RL **transients** 暂态 with their time constants.
- Know what cannot change suddenly, and what happens long after a switch.
- Use complex **impedance** 阻抗, **reactance** 电抗 and **phase** 相位.
- Find the resonant frequency, current and **quality factor** 品质因数.

# RC transients

- From zero charge: $V_C = V_s(1 - e^{-t/\tau})$ and $I = (V_s/R)e^{-t/\tau}$.
- After one $\tau$, the capacitor has reached 63% of its final voltage; after $5\tau$, more than 99%.

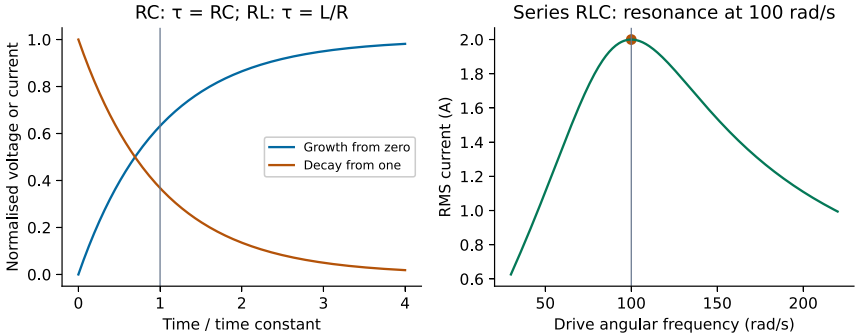
# RL transients

- From zero current: $I = (V_s/R)(1 - e^{-t/\tau})$.
- $L = 0.40$ H and $R = 2.0\ \Omega$ give $\tau = 0.20$ s.

# What cannot jump

- A capacitor's voltage and an inductor's current cannot change instantly.
- Just after a switch closes, an uncharged capacitor acts like a wire and an inductor with no current acts like a break.
- Long afterwards, the capacitor blocks steady current (open circuit) and the inductor passes it freely (wire). The stored energies $\frac{1}{2}CV^2$ and $\frac{1}{2}LI^2$ end as heat in the resistors.

# Figure: What cannot jump




- $\tau = \mathrm{RC}$ or $\mathrm{L/R}$; a series RLC circuit draws its largest current at resonance
- Read the axes and units first.

# Impedance and phase

- The current lags the voltage when $X > 0$ (inductive) and leads it when $X < 0$ (capacitive).
- Never add $R$, $\omega L$ and $1/\omega C$ as plain numbers; they are at right angles.

$$Z = R + iX, \qquad X = \omega L - \frac{1}{\omega C}, \qquad |I| = \frac{|V|}{|Z|}, \qquad \tan\varphi = \frac{X}{R}$$

# Resonance

- Only the resistor takes power: $P = I_{\text{rms}}^2 R$.
- The **quality factor** $Q = \omega_0 L/R$ measures sharpness: the resonance peak is about $\omega_0/Q$ wide.
- The free, damped oscillation frequency $\sqrt{1/LC - (R/2L)^2}$ is a different quantity.

# Worked example: charging a capacitor

- $\tau = RC = 2.0$ s, and the starting current is $12/2000 = 6.0$ mA.
- At $t = \tau$: $V_C = 12(1 - e^{-1}) = 7.6$ V and $I = 6.0e^{-1} = 2.2$ mA.
- Energy finally stored: $\frac{1}{2}(1.0 \times 10^{-3})(12)^2 = 72$ mJ; the resistor turns another 72 mJ into heat while charging.

# Worked example: a series RLC circuit

- $\omega_0 = 1/\sqrt{(0.10)(1.0\times10^{-3})} = 100$ rad/s. There $I_{\text{rms}} = 20/10 = 2.0$ A and $P = 40$ W. $Q = (100)(0.10)/10 = 1$.
- At $\omega = 200$ rad/s: $X = 20 - 5 = 15\ \Omega$, $|Z| = \sqrt{10^2 + 15^2} = 18\ \Omega$, so $I_{\text{rms}} = 1.1$ A.
- There $X > 0$, so the current lags by $\tan^{-1}(1.5) = 56°$.

# Common errors

- Letting a capacitor's voltage or an inductor's current jump at a switch.
- Using $\tau = RL$ or $\tau = R/L$; it is $L/R$.
- Adding reactances and resistance as plain numbers.

# Common errors (2)

- Getting the sign of the phase wrong: inductive means current lags.
- Confusing the driven resonance with the free damped frequency.

# Key terms

- **resonance**: The driving frequency at which a circuit's reactances cancel and the current is largest.
- **transient**: The short-lived change in a circuit after a switch, before it settles.
- **impedance**: The AC opposition to current, $Z = R + iX$.
- **reactance**: The part of the impedance from inductors and capacitors, $\omega L - 1/\omega C$.

# Check yourself

- A 10 V source drives an RL circuit with $L = 0.40$ H and $R = 2.0\ \Omega$. What is the current 0.20 s after switching on?
- What is the phase between current and voltage at resonance?
- Just after a switch closes, how does an uncharged capacitor behave?