

RC and RL transients, impedance and resonance

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A capacitor voltage and an inductor current cannot change instantly under finite ordinary circuit drives, but their steady-state behaviour is different. Solve RC charging and discharging with stated initial conditions Solve RL current response and account for stored energy Use complex impedance, phase and resonance in sinusoidal circuits

Solve the capacitor response

Reasoning

For a resistor R in series with a capacitor C and a DC source V_s , Kirchhoff's voltage law is $R \, dq/dt + q/C = V_s$. Define $V_C = q/C$ and $\tau = RC$. After a switch to a constant source, $V_C(t) = V_s + (V_C(0) - V_s)\exp(-t/\tau)$. An initially uncharged capacitor therefore has $V_C = V_s(1 - \exp(-t/\tau))$ and charging current $I = (V_s/R)\exp(-t/\tau)$. With the source removed and the resistor connected across it, $V_C = V_C(0)\exp(-t/\tau)$. Capacitor voltage is continuous across the switch if there is no impulsive current; the resistor current may change instantly.

Track inductor current and energy

Reasoning

In a series RL circuit, Kirchhoff's law gives $L \, di/dt + Ri = V_s$ and $\tau = L/R$. The current after a constant drive is $i(t) = V_s/R + (i(0) - V_s/R)\exp(-t/\tau)$. Inductor current is continuous for a finite voltage; its voltage may change abruptly as switching changes di/dt . At late times an ideal inductor in a DC circuit acts as a zero-voltage connection, while an ideal capacitor has zero DC current. Stored energies are $LI^2/2$ and $CV_C^2/2$. During decay into a resistor, the initially stored energy becomes resistor heat rather than disappearing when the source is disconnected.

Use the phasor convention

Reasoning

Use the declared phasor convention $\exp(i\omega t)$. A resistor has impedance R , an inductor $i\omega L$ and a capacitor $1/(i\omega C) = -i/(\omega C)$. A series RLC circuit therefore has $Z = R + iX$ with $X = \omega L - 1/(\omega C)$. Divide the source voltage phasor by Z to obtain current. Its magnitude is $V_{\text{rms}}/\sqrt{R^2 + X^2}$ when rms quantities are used. The impedance phase ϕ satisfies $\tan\phi = X/R$; current lags voltage for $X > 0$ and leads it for $X < 0$. Adding the scalar magnitudes R , ωL and $1/(\omega C)$ loses the vector phase information.

Separate resonance from decay

Reasoning

Series resonance occurs at $\omega_0 = 1/\sqrt{LC}$ when $X=0$. Current is maximal for a fixed voltage in this ideal series model, and the voltage/current phase difference is zero. Mean real power is $V_{\text{rms}} I_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$; ideal L and C exchange energy but dissipate no average power. This differs from transient natural frequency: a damped series circuit can oscillate at $\sqrt{1/(LC) - (R/(2L))^2}$ when underdamped. Large resistance removes such free oscillation but does not change the condition $X=0$ of the ideal driven series impedance. State whether a question asks for a step response, free decay or sinusoidal steady state.

Worked reasoning 1

Reasoning

An RC circuit switches to fixed source voltage V_s . Charge conservation and Kirchhoff's law give

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$RC\dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s]e^{-t/(RC)}.$$

Worked reasoning 3

Reasoning

For $R = 2$ kilohms, $C = 1$ millifarad, $V_s = 12$ V and initial voltage zero,

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$\tau = RC = (2000\ \Omega)(0.001\ \text{F}) = 2.0\ \text{s}.$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

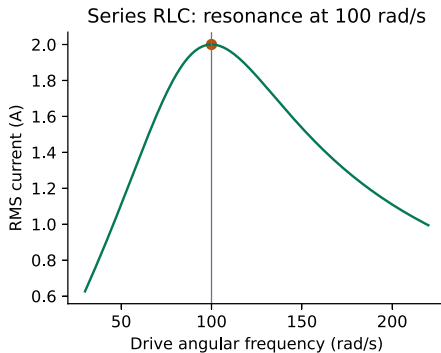
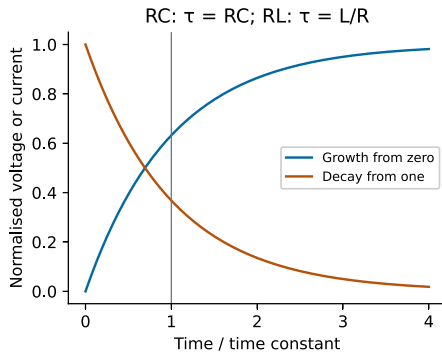
$$V_C(\tau) = V_s(1 - e^{-1}) = (12 \text{ V})(1 - e^{-1}) = 7.59 \text{ V}.$$

Worked reasoning 7

Reasoning

The time constant controls the remaining difference from the final state.

Read the original diagram



- Original GRE teaching diagram: RC and RL transients, impedance and resonance
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

An RC capacitor starts at 8 V and switches to a -4 V ideal source through R. Find its voltage at arbitrary t , the first zero crossing and the initial current defined into its positive-voltage plate. Explain voltage continuity. **[5]**

Independent transfer · reasoning

Reasoning

With $\tau = RC$, $V_C = -4 + 12e^{-t/\tau}$ volts. Zero crossing satisfies $e^{-t/\tau} = 1/3$, so $t = \tau \ln 3$. $I(0) = (V_s - V_C(0))/R = -12\text{ V}/R$. Finite current cannot change capacitor charge instantaneously, so voltage begins at 8 V. Award solution, endpoint check, crossing, signed current and continuity.

Check assumptions and limits

Reasoning

RC has $\tau=RC$, while RL has $\tau=L/R$. Do not confuse current amplitude with rms current, or resonance of a driven impedance with damped free-oscillation frequency.

Keep the technical distinctions

Reasoning

time constant : The exponential response scale, RC for an RC circuit and L/R for an RL circuit. impedance : The complex voltage-to-current phasor ratio in sinusoidal steady state.

Check yourself

- Define time constant and give its condition of use.
- Define impedance and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.