Polarisation, magnetisation and material fields

GRE undergraduate preparation · local supported objectives

Polarisation, magnetisation and material fields

- Use $\mathbf{P}$, $\mathbf{D}$ and **bound charge** 束缚电荷.
- Decide whether charge or voltage is fixed when a dielectric is inserted.
- Apply the field conditions at an interface.
- Use **magnetisation** 磁化强度, $\mathbf{H}$, $\mathbf{B}$ and the **susceptibility** 磁化率.

Polarisation and the D field

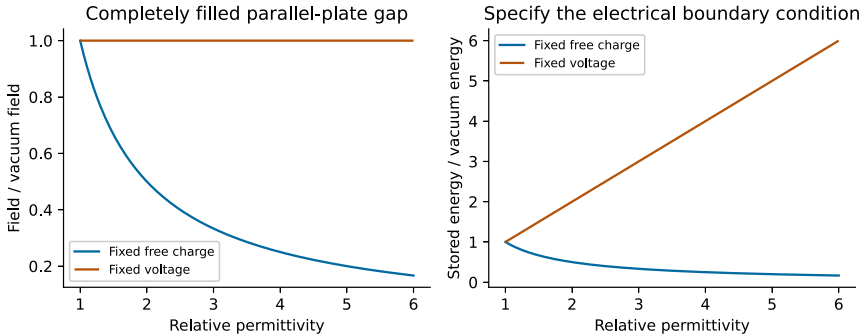
- **Bound charge**: $\rho_b = -\nabla \cdot \mathbf{P}$ inside and $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$ on surfaces.
- In a linear dielectric, $\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$. The field inside is reduced by ε_r, not removed.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \qquad \nabla \cdot \mathbf{D} = \rho_{\text{free}}$$

Fixed charge or fixed voltage

- Disconnected (fixed Q): V, E and the energy $U = Q^2/2C$ all fall by ε_r.
- Connected (fixed V): Q and the energy $U = \frac{1}{2}CV^2$ rise by ε_r; $E = V/d$ is unchanged.
- The same slab gives opposite energy changes in the two cases.

Figure: Fixed charge or fixed voltage



- At fixed charge a dielectric lowers E and energy by κ; at fixed voltage the energy rises by κ
- Read the axes and units first.

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Interfaces

- The normal component of $\mathbf{D}$ jumps by the free surface charge (zero if there is none).
- The tangential component of $\mathbf{E}$ is continuous.
- So when ε changes, the normal component of $\mathbf{E}$ changes, and field lines bend.

Magnetisation, H and B

- In a long solenoid, $H = nI$ whatever the core; the core changes B through M.
- Diamagnets have a small negative χ_m; paramagnets a small positive one.
- Ferromagnets have huge, non-linear magnetisation, saturate, and show **hysteresis** **磁滞**: their magnetisation depends on their history.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \qquad \mathbf{M} = \chi_m \mathbf{H} \text{ (linear materials)}$$

Worked example: a filled capacitor

- $D = \sigma$, so $E = \sigma/(4\varepsilon_0)$, a quarter of the empty value.
- $P = \varepsilon_0(\varepsilon_r - 1)E = \frac{3}{4}\sigma$; the **bound charge** $-\frac{3}{4}\sigma$ sits next to the positive plate.
- The voltage and stored energy both fall to a quarter.

Worked example: battery on or off

- Disconnected: $C = 40$ pF, $V = 25$ V, $U = 12.5$ nJ. The slab is pulled in, and the field does work on it.
- Connected: $Q = 4.0$ nC and $U = 200$ nJ. The battery supplies $QV = 300$ nJ of extra energy: 150 nJ is stored, and 150 nJ is the work done pulling the slab in.
- A solenoid with $H = 400$ A/m and a core of $\chi_m = 0.5$: $M = 200$ A/m and $B = \mu_0(600) = 0.75$ mT.

Common errors

- Saying the field inside a dielectric is zero; that is a conductor.
- Assuming the energy always falls when a dielectric is inserted.
- Using total charge in $\nabla \cdot \mathbf{D}$; it counts free charge only.

Common errors (2)

- Taking tangential $\mathbf{D}$ or normal $\mathbf{E}$ as continuous at an interface.
- Treating a ferromagnet as linear.

Key terms

- **permittivity**: How strongly a material reduces an electric field; $\varepsilon = \varepsilon_r \varepsilon_0$.
- **dielectric**: An insulator that becomes polarised in an electric field.
- **polarisation**: Electric dipole moment per unit volume.
- **bound charge**: Charge that appears on or in a dielectric because of its polarisation.

Check yourself

- A disconnected capacitor is filled with a slab of $\varepsilon_r = 3$. What happens to its voltage?
- A diamagnet has $\chi_m = -1.0 \times 10^{-5}$ in $H = 1000$ A/m. What is M?
- Is the normal component of $\mathbf{D}$ continuous at an interface with no free charge?