

# Polarisation, magnetisation and material fields

GRE undergraduate preparation · local supported objectives

## Choose the physical question

### Reasoning

Putting a dielectric into a capacitor changes the field differently depending on whether its battery remains connected. Relate free and bound charge to  $D$ ,  $E$  and polarisation  
Compare fixed-charge and fixed-voltage dielectric changes Use magnetic constitutive response and free-current boundary conditions

## Separate free and bound charge

### Reasoning

Electric polarisation  $\mathbf{P}$  is electric dipole moment per unit volume, measured in  $\text{C}/\text{m}^2$ . Define  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$  so that  $\nabla \cdot \mathbf{D} = \rho_{\text{free}}$ ; the total charge, including bound charge, still appears in  $\nabla \cdot \mathbf{E} = \rho_{\text{total}}/\epsilon_0$ . Bound volume charge is  $\rho_{\text{b}} = -\nabla \cdot \mathbf{P}$  and bound surface charge is  $\sigma_{\text{b}} = \mathbf{P} \cdot \mathbf{n}$ , where  $\mathbf{n}$  points outward from the material. Uniform  $\mathbf{P}$  therefore gives no bound charge in the bulk but can give surface charges of opposite sign. A dielectric is not an ideal metal: polarisation need not make its internal electric field zero.

## Apply the material relation

### Reasoning

For a linear isotropic dielectric,  $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$  and  $\mathbf{D} = \epsilon \mathbf{E}$  with  $\epsilon = \epsilon_0(1 + \chi_e) = \epsilon_0 \epsilon_r$ . These simple scalar relations assume the response is linear and ignore anisotropy, strong dispersion and nonlinear effects. In a fully filled, large parallel-plate capacitor with negligible edge effects,  $\mathbf{D}$  normal to the plates equals the free surface-charge density. If  $\sigma_{\text{free}}$  stays fixed, increasing  $\epsilon$  reduces  $E = \sigma_{\text{free}} / \epsilon$ . The bound charges oppose the applied field; their magnitude is not automatically equal to the free plate charge.

## Hold charge or voltage fixed

### Reasoning

For plate area  $A$  and spacing  $d$ ,  $C = \epsilon A/d$ . Disconnecting the battery fixes free charge  $Q$ :  $V = Q/C$  falls and stored energy  $U = Q^2/(2C)$  falls when a dielectric increases  $C$ .

Leaving an ideal voltage source connected fixes  $V$ :  $Q = CV$  and stored energy  $U = CV^2/2$  both rise. The source supplies energy and moving the dielectric can involve mechanical work; compare the stated electrical energy quantity rather than assuming the capacitor alone is an isolated system. At a material interface  $\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_{\text{free}}$ , whereas static tangential  $\mathbf{E}$  is continuous. Normal  $\mathbf{E}$  generally changes when permittivity changes.

## Distinguish magnetic fields

### Reasoning

Magnetisation  $M$  is magnetic dipole moment per unit volume, measured in A/m. In SI,  $B = \mu_0(H + M)$ ; for a linear isotropic response  $M = \chi_m H$  and  $B = \mu_0(1 + \chi_m)H$ .  $B$  and  $H$  have different units and roles. A long uniform solenoid has  $H \approx nI$  when end and demagnetising effects are negligible; material response then changes  $B$ . Free surface current sets  $n \times (H_2 - H_1) = K_{\text{free}}$ , and normal  $B$  remains continuous. Ferromagnetic hysteresis and saturation cannot be represented by one constant  $\chi_m$  over every field.

## Worked reasoning 1

### Reasoning

A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance becomes four times its original value.

## Worked relation 2

### Reasoning

Apply this relation only under the conditions just stated.

$$V' = Q/(4C) = V/4, \quad U' = Q^2/(2 \times 4C) = U/4.$$

## Worked reasoning 3

### Reasoning

If an ideal battery stays connected instead,  $V$  is fixed:

## Worked relation 4

### Reasoning

Apply this relation only under the conditions just stated.

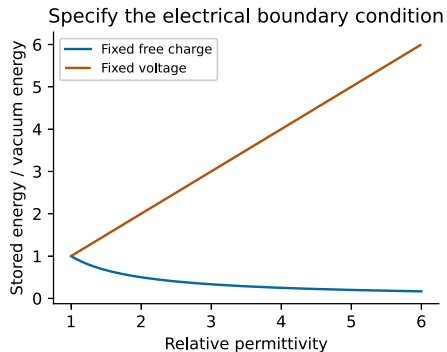
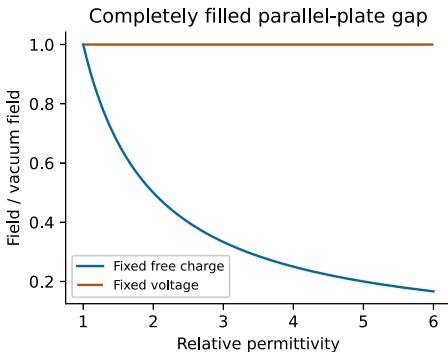
$$Q' = 4CV = 4Q, \quad U' = \frac{1}{2}(4C)V^2 = 4U.$$

## Worked reasoning 5

### Reasoning

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.

## Read the original diagram



- Original GRE teaching diagram: Polarisation, magnetisation and material fields
- Identify the axes, units, directions and the stated model.

## Independent transfer · attempt

### Reasoning

A slab fills half the thickness of a parallel-plate capacitor and has permittivity  $3\epsilon_0$ ; the remaining thickness is vacuum. Find  $C$  relative to the empty capacitor, and the two normal fields at fixed free charge  $Q$ . Ignore edge effects. **[5]**

# Independent transfer · reasoning

## Reasoning

Layers are in series. With no free interface charge,  $D = Q/A$  is common.

$E_{vac} = Q/(\epsilon_0 A)$  and  $E_d = Q/(3\epsilon_0 A)$ .  $V = (d/2)(E_{vac} + E_d) = 2Qd/(3\epsilon_0 A)$ , hence  $C = 3\epsilon_0 A/(2d) = 3C_0/2$ . Award common  $D$ , each field, voltage sum and capacitance. Normal  $E$  is discontinuous;  $D$  is continuous.

## Check assumptions and limits

### Reasoning

Use free charge in the  $D$  equation and total charge in the  $E$  equation. Do not confuse fixed  $Q$  with fixed  $V$ , or identify  $B$  numerically with  $H$ .

## Keep the technical distinctions

### Reasoning

electric polarisation : Electric dipole moment per unit volume,  $P$ , contributing bound charge. magnetisation : Magnetic dipole moment per unit volume,  $M$ , entering  $B = \mu_0(H + M)$ .

## Check yourself

- Define electric polarisation and give its condition of use.
- Define magnetisation and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.