Fluid continuity, pressure energy and viscous flow

GRE undergraduate preparation · local supported objectives

Fluid continuity, pressure energy and viscous flow

- Use **continuity 连续性** and **hydrostatic pressure 静水压强**.
- Apply Bernoulli's equation along a **streamline 流线**, and know its conditions.
- Use Poiseuille's law for viscous flow.
- Judge **laminar** or turbulent flow from the **Reynolds number**.

Continuity and hydrostatics

- Halving the radius quarters the area and multiplies the speed by 4.
- In a fluid at rest, the pressure at depth h is $p = p_0 + \rho g h$.
- Use absolute or gauge pressure consistently. Every 10 m of water adds about one atmosphere.

Bernoulli's equation

- Each term is an energy per unit volume.
- Faster flow at the same height means lower pressure (the Venturi effect).
- Water leaving a hole at depth h below an open surface has $v = \sqrt{2gh}$ (Torricelli's law).

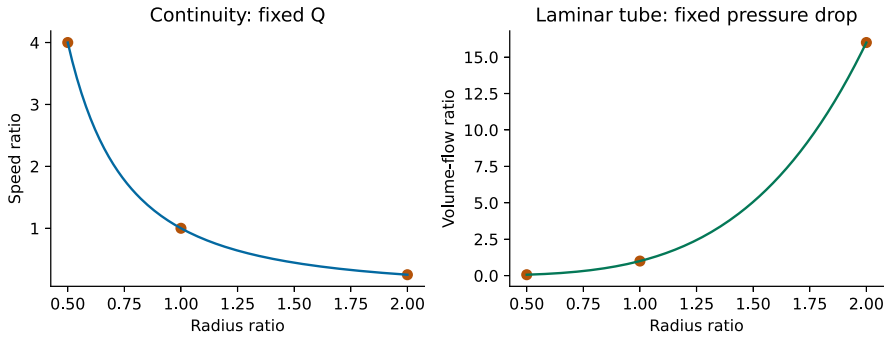
$$p + \tfrac{1}{2}\rho v^2 + \rho g z = \text{constant}$$

Viscous flow: Poiseuille's law

- Doubling R at the same Δp multiplies the flow by 16.
- This compares different tubes at the same pressure drop; continuity compares parts of one pipe at the same Q.

$$Q = \frac{\pi R^4 \, \Delta p}{8 \eta L}$$

Figure: Viscous flow: Poiseuille's law



- Continuity gives v $\propto 1/r^2$; Poiseuille flow gives Q $\propto r^4$
- Read the axes and units first.

The Reynolds number

- It compares inertia with viscosity and has no units.
- Low Re: viscous, **laminar** flow. High Re: inertia dominates and turbulence becomes possible. For a round pipe the change is near Re $\approx$ 2000, but the threshold depends on the geometry.
- Poiseuille's R^4 law fails once the flow is turbulent.

$$\mathrm{Re} = \frac{\rho v D}{\eta}$$

Worked example: a narrowing pipe

- Continuity: $v_2 = 4 \times 1.0 = 4.0$ m/s and $Q = 4.0 \times 10^{-4}$ m^3/s.
- Bernoulli: $p_1 - p_2 = \frac{1}{2}(1000)(4.0^2 - 1.0^2) = 7500$ Pa.
- With viscosity, the pressure would also fall along each straight section.

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Worked example: a narrowed artery and a water pipe

- An artery's radius shrinks by 20% at the same pressure difference: Q falls to $0.8^4 = 0.41$ of its old value, a drop of 59%.
- Water ($\eta = 1.0 \times 10^{-3}$ Pa s) at 0.50 m/s in a 2.0 cm pipe: $\mathrm{Re} = (1000)(0.50)(0.020)/(1.0 \times 10^{-3}) = 10{,}000$.
- That is well above 2000, so the flow is probably turbulent and Poiseuille's law does not apply.

Common errors

- Using Bernoulli's equation across a pump or with strong viscous losses.
- Confusing continuity (Q fixed) with Poiseuille (Δp fixed).
- Mixing gauge and absolute pressure.

Common errors (2)

- Using the radius instead of the diameter in the **Reynolds number**.
- Applying the R^4 law to turbulent flow.

Key terms

- **laminar**: Flowing in smooth layers that do not mix.
- **turbulent**: Flowing with irregular swirls and mixing.
- **viscosity**: A fluid's internal friction, its resistance to flowing.
- **Reynolds number**: The dimensionless ratio of inertial to viscous effects in a flow.

Check yourself

- Water leaves a hole 5.0 m below the surface of an open tank. How fast?
- What is the gauge pressure 10 m below the surface of water?
- Within one pipe, the radius doubles. What happens to the speed?