

Fluid continuity, pressure energy and viscous flow

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A narrow pipe carries the same steady flow rate as a wide pipe, but its pressure behaviour depends on both speed and viscous losses. Combine incompressible continuity with pressure, height and kinetic energy. Apply hydrostatic and Bernoulli relations only under their stated conditions. Use viscosity, Poiseuille scaling and Reynolds number to distinguish flow regimes.

Conserve the flow rate

Reasoning

For steady flow, the mass passing successive cross-sections per unit time is equal: ρAv is constant. An incompressible liquid has nearly constant ρ , so $Q=Av$ is the volume flow rate. Halving pipe radius quarters area and multiplies mean speed by four for the same Q . Pressure does not determine Q without a model of the rest of the system. In a stationary liquid, $dp/dz=-\rho g$ with z upward; therefore pressure increases by ρgh a distance h below a surface. Use absolute or gauge pressure consistently on both sides.

Account for pressure energy

Reasoning

For steady, incompressible, inviscid motion along a streamline with no pump or dissipative loss, Bernoulli gives $p + \rho v^2/2 + \rho g z = \text{constant}$. These three terms are energy per volume and have pressure units. At equal height, larger speed requires smaller static pressure under these assumptions. A higher outlet also uses pressure/kinetic energy to gain gravitational energy. Bernoulli along one streamline need not imply the same constant on different streamlines in rotational flow. A stagnation point has $v=0$ and converts local speed energy to a pressure rise in the ideal model.

Include viscous loss

Reasoning

Viscosity transports momentum between neighbouring fluid layers. For steady fully developed laminar flow of a Newtonian incompressible liquid in a circular tube, $Q = \pi R^4 \Delta p / (8 \eta L)$. Here Δp is the pressure drop along the tube, η dynamic viscosity and L tube length. Doubling radius at fixed Δp , η and L multiplies Q by sixteen. Doubling Q at fixed geometry needs twice the pressure drop. This viscous pressure loss cannot be added to a loss-free Bernoulli equation as though nothing changes; include a dissipative pressure/head loss or use the viscous model.

Check the flow regime

Reasoning

The Reynolds number $Re = \rho v D / \eta$ compares inertial and viscous effects using characteristic speed v and length D . For a pipe use mean speed and internal diameter, not radius. Re is dimensionless: density times speed times length has the same units as dynamic viscosity. Small Re favours viscous dominance; transition to turbulence depends on geometry and disturbances, so a single threshold is not a universal law. Poiseuille scaling is not safe after assuming a turbulent flow. Check volume continuity, sign of pressure change and the regime before selecting a formula.

Worked reasoning 1

Reasoning

A steady incompressible horizontal ideal flow has areas 4 and 1 square centimetres. At the first section speed is 1 m/s. Continuity and Bernoulli give

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$v_2 = A_1 v_1 / A_2 = (4 \text{ cm}^2)(1 \text{ m/s}) / (1 \text{ cm}^2) = 4 \text{ m/s}.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

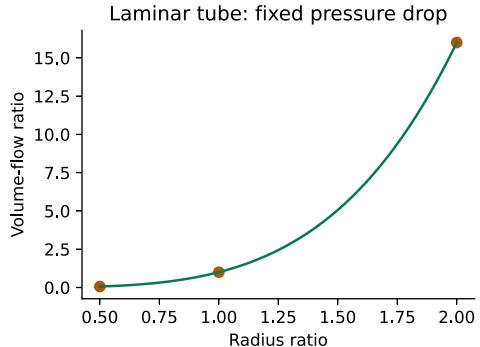
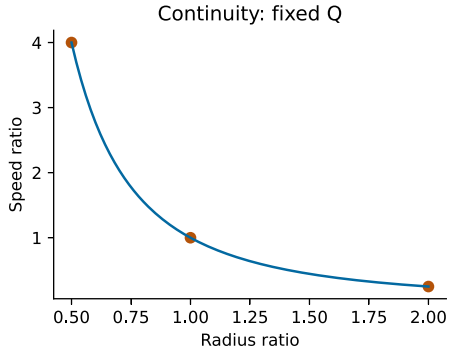
$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(1000 \text{ kg/m}^3)[(4 \text{ m/s})^2 - (1 \text{ m/s})^2] = 7500 \text{ Pa}.$$

Worked reasoning 5

Reasoning

These relations assume no pump, viscosity loss or height change. A real narrow tube may violate them.

Read the original diagram



- Original GRE teaching diagram: Fluid continuity, pressure energy and viscous flow
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Two fully developed laminar Newtonian tubes of equal length and viscosity connect in series. Their radii are R and $2R$. Find their pressure-drop ratio and equivalent hydraulic resistance. Compare with connecting the same tubes in parallel. **[5]**

Independent transfer · reasoning

Reasoning

Define resistance $\mathcal{R} = \Delta p/Q = 8\eta L/(\pi R^4)$ for the narrow tube. Wide-tube resistance is $\mathcal{R}/16$. In series Q is common, pressure ratio is 16:1, and total resistance $17\mathcal{R}/16$. In parallel pressure is common, flow ratio narrow:wide is 1:16 and equivalent resistance $\mathcal{R}/17$. Award radius scaling, series ratio, series total, parallel ratio and parallel total.

Check assumptions and limits

Reasoning

Continuity keeps Q constant across one steady pipe; Poiseuille compares Q between different systems at a specified pressure drop. State what is held fixed before comparing radius powers.

Keep the technical distinctions

Reasoning

volume flow rate : Volume crossing a section per unit time, $Q=Av$ for mean speed v .
dynamic viscosity : The coefficient relating shear stress to velocity gradient in a Newtonian fluid.

Check yourself

- Define volume flow rate and give its condition of use.
- Define dynamic viscosity and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.