Accelerating and rotating reference frames

GRE undergraduate preparation · local supported objectives

Accelerating and rotating reference frames

- Handle a frame that accelerates in a straight line.
- Use the Coriolis, **centrifugal** 离心的 and Euler terms in a **rotating frame** 旋转参考系.
- Get the directions of these terms right.
- Check that the frame terms vanish when the frame stops accelerating.

Translating frames

- In a lift accelerating upward at a, the floor pushes with $N = m(g + a)$.
- This is the same physics as the ground-frame equation $N - mg = ma$, only written from inside the lift.

$$m\mathbf{a}' = \mathbf{F}_{\text{real}} - m\mathbf{A}$$

Rotating frames

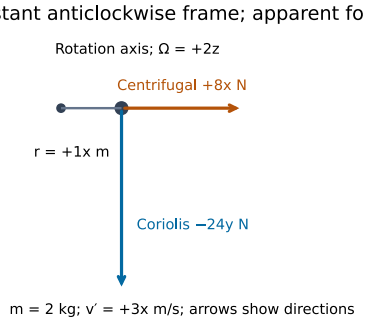
- Coriolis term $-2m\Omega \times \mathbf{v}'$: needs a velocity $\mathbf{v}'$ relative to the rotating frame.
- Centrifugal term: points outward from the axis, with size $m\Omega^2 r_\perp$.
- **Euler force 欧拉力** $-m\dot{\Omega} \times \mathbf{r}$: only when the rotation rate changes.

$$m\mathbf{a}' = \mathbf{F} - 2m\Omega \times \mathbf{v}' - m\Omega \times (\Omega \times \mathbf{r}) - m\dot{\Omega} \times \mathbf{r}$$

Getting directions right

- Centrifugal: along $+x$, away from the axis.
- $\Omega \times \mathbf{v}'$ points along $+y$, so the Coriolis force points along $-y$: to the right of the motion.
- On the northern half of the Earth, moving bodies are deflected to the right; on the southern half, to the left.

Figure: Getting directions right



- Centrifugal force $m\Omega^2 r = 8$ N; Coriolis force $-2m\Omega \times v' = -24$ N on y
- Read the axes and units first.

The rotating Earth

- At latitude λ, a Foucault pendulum's plane turns once in $24\ \mathrm{h}/\sin\lambda$: once a day at a pole, never at the equator.
- The centrifugal term makes the effective g at the equator about 0.034 $\mathrm{m/s^2}$ smaller than it would otherwise be.

Checks

- A body at rest on a steady platform has no Coriolis force; the real force on it is $m\Omega^2 r$ inward.
- The Coriolis force is perpendicular to $\mathbf{v}'$, so it does no work.
- Set $\Omega \to 0$ and $\mathbf{A} \to 0$: ordinary Newtonian motion must come back.

Worked example: a lift and a platform

- A 10 kg passenger in a lift accelerating upward at 2.0 m/s^2 ($g = 10$): $N = 10(10 + 2.0) = 120$ N.
- A 2.0 kg particle 1.0 m from the axis of a platform turning at 2.0 rad/s, moving outward at 3.0 m/s: centrifugal $2.0 \times 2.0^2 \times 1.0 = 8.0$ N along $+x$.
- Coriolis: $2 \times 2.0 \times 2.0 \times 3.0 = 24$ N along $-y$. With no real force, $\mathbf{a}' = (4.0, -12)$ m/s^2.

Worked example: Foucault's pendulum in Paris

- Period of the plane's rotation: $24\ \text{h} / \sin 48.9^\circ = 24 / 0.753 = 31.9\ \text{h}$.
- Rate: $360^\circ / 31.9\ \text{h} = 11.3^\circ$ per hour, clockwise seen from above.
- At the North Pole the plane would turn once in a day (strictly, a sidereal day of 23.9 h).

Common errors

- Including a Coriolis force on a body at rest in the rotating frame.
- Using the velocity relative to the ground instead of relative to the rotating frame.
- Getting the Coriolis direction backwards; work out $\Omega \times \mathbf{v}'$ and then change its sign.

Common errors (2)

- Adding centrifugal force in an **inertial frame**.
- Forgetting the $\sin\lambda$ factor for a Foucault pendulum.

Key terms

- **apparent force**: A term $-m\mathbf{A}$ or a rotation term that appears only in a non-inertial frame.
- **Coriolis force**: The rotating-frame term $-2m\Omega \times \mathbf{v}'$ on a moving body.
- **centrifugal**: Directed outward, away from the axis of rotation.
- **rotating frame**: A reference frame that turns relative to an **inertial frame**.

Check yourself

- A 60 kg person stands in a lift accelerating downward at 2.0 $\mathrm{m/s^2}$ ($g = 10$). What does the floor push with?
- How long does a Foucault pendulum take to turn once at the equator?
- What is the Coriolis force on a particle at rest in a rotating frame?