

Accelerating and rotating reference frames

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A person standing still on a turning platform needs a real inward force, although their platform-relative acceleration is zero. Transform acceleration with translating and rotating frame terms Determine centrifugal, Coriolis and Euler directions from cross products Separate apparent forces from real interactions and test inertial limits

Translate the frame

Reasoning

In a translating frame with origin acceleration A , Newton's equation becomes $m a' = F_{\text{real}} - mA$. An upward-accelerating elevator therefore has $N - mg = ma$ in the ground frame, or $N - mg - ma = 0$ for its stationary passenger in the elevator frame. These are the same prediction. The additional term is an apparent force caused by the chosen accelerating coordinates; it is not a new contact with another body. Uniform translation with $A=0$ changes velocity but needs no apparent force.

Differentiate rotating axes

Reasoning

For a rotating basis, differentiating a vector adds $\Omega \times$ that vector. Applying this twice gives $\mathbf{a} = \mathbf{a}_{\text{origin}} + \mathbf{a}' + 2\Omega \times \mathbf{v}' + \Omega \times (\Omega \times \mathbf{r}) + \dot{\Omega} \times \mathbf{r}$. Here $\mathbf{r}$ and $\mathbf{v}'$ are measured relative to the moving origin in its rotating axes; all vectors in a calculation must be expressed in the same basis at the same instant. Move the last three rotational terms to the force side to obtain Coriolis $-2m\Omega \times \mathbf{v}'$, centrifugal $-m\Omega \times (\Omega \times \mathbf{r})$, and Euler $-m\dot{\Omega} \times \mathbf{r}$. A constant rotation removes the Euler term, not the Coriolis term.

Check cross-product directions

Reasoning

Take Ω along $+z$, an anticlockwise platform viewed from above, and a particle at r along $+x$. Centrifugal force is along $+x$ with magnitude $m\Omega^2 r$. If the particle moves outward with v' along $+x$, then $\Omega \times v'$ is $+y$, so Coriolis force points $-y$. Reversing the relative velocity reverses Coriolis; keeping $v'=0$ makes it vanish. Coriolis is perpendicular to v' and does no instantaneous work on that relative motion. Centrifugal force is outward from the rotation axis, not necessarily outward from the chosen origin in an arbitrary three-dimensional position.

Balance real and apparent forces

Reasoning

A body at rest on the platform has $a'=v'=0$. With constant rotation and a fixed origin, its real force must be $m\Omega\times(\Omega\times r)$, inward, cancelling the outward apparent term in the rotating equation. If rotation changes, a tangential real force must also balance the Euler term. Always check $\Omega\rightarrow 0$ and $A\rightarrow 0$: ordinary inertial Newtonian motion must return. Do not mix the real inward centripetal requirement with an added outward real reaction on the same body; interaction partners belong in separate free-body diagrams.

Worked reasoning 1

Reasoning

In a rotating frame, Coriolis force uses the relative velocity.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$\mathbf{F}_C = -2m\boldsymbol{\Omega} \times \mathbf{v}', \quad \mathbf{F}_{cf} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

Worked reasoning 3

Reasoning

For $m = 2$ kg, $\Omega = 2$ rad/s along z , $r = 1$ m along x and $v' = 3$ m/s along x ,

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$F_{cf} = m\Omega^2 r = (2 \text{ kg})(2 \text{ rad/s})^2(1 \text{ m}) = 8 \text{ N} \quad (+x).$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

$$F_C = 2m\Omega v = 2(2\text{ kg})(2\text{ rad/s})(3\text{ m/s}) = 24\text{ N} \quad (-y).$$

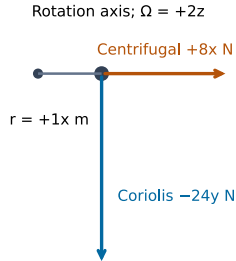
Worked reasoning 7

Reasoning

Euler force is zero only because Ω is constant. These are coordinate terms, not extra physical contacts.

Read the original diagram

Constant anticlockwise frame; apparent forces



$m = 2 \text{ kg}$; $v' = +3x \text{ m/s}$; arrows show directions

- Original GRE teaching diagram: Accelerating and rotating reference frames
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A particle is instantaneously at rest relative to a platform at radius r along positive x . The platform has angular velocity Ω along positive z and angular acceleration α along positive z . Find the real force needed to keep the particle fixed on the platform. **[4]**

Independent transfer · reasoning

Reasoning

With $\mathbf{v}' = \mathbf{a}' = 0$, the inertial acceleration is $\Omega \times (\Omega \times \mathbf{r}) + \dot{\Omega} \times \mathbf{r} = -\Omega^2 r\hat{x} + \alpha r\hat{y}$. Real force is $m(-\Omega^2 r\hat{x} + \alpha r\hat{y})$. Coriolis vanishes, while the Euler apparent force points -y and centrifugal points +x. Award centripetal term, tangential term, real force and apparent-force distinction.

Check assumptions and limits

Reasoning

The relative velocity v' belongs in the Coriolis term. Using the full inertial velocity counts rotation twice.

Keep the technical distinctions

Reasoning

Coriolis force : The apparent rotating-frame force $-2m\Omega \times v'$ caused by relative motion. centrifugal force : The apparent rotating-frame force $-m\Omega \times (\Omega \times r)$, directed away from the rotation axis.

Check yourself

- Define Coriolis force and give its condition of use.
- Define centrifugal force and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.