Nuclear binding, particle families and Hall carriers

GRE undergraduate preparation · local supported objectives

Nuclear binding, particle families and Hall carriers

- Compare stability by **binding energy per nucleon** 平均核子结合能.
- Find reaction energies (**Q value** Q 值).
- Sort particles into **leptons** 轻子, **mesons** 介子 and **baryons** 重子 by their **quarks** 夸克.
- Use the Hall coefficient to find the sign and density of carriers.

Binding energy per nucleon

- B/A rises steeply for light nuclei, peaks near iron and nickel (about 8.8 MeV), and falls slowly for heavy ones.
- $B = 24$ MeV with $A = 4$ gives 6 MeV per nucleon; $B = 80$ MeV with $A = 10$ gives 8 MeV.
- Fusion of light nuclei and fission of heavy ones both move towards the peak and release energy.

Reaction energies

- Positive Q releases energy, shared as kinetic energy of the products.
- With atomic masses, the electron masses cancel as long as the charge balances.

$$Q = \left(m_{\text{initial}} - m_{\text{final}}\right) c^2$$

Particle families

- **Leptons** (electron, muon, tau and their neutrinos) contain no **quarks**. The muon is a heavy lepton, not a meson.
- **Mesons** are a quark and an antiquark: pions, kaons.
- Baryons are three quarks: proton *uud*, neutron *udd*.

Figure: Particle families

Particle families in the ordinary constituent model

Examples	Ordinary constituent model	Family
Electron / muon	None	Lepton
Pion / kaon	Quark + antiquark	Meson
Proton / neutron	Three quarks	Baryon

- Classify a particle by its constituents: **leptons**, **mesons** or **baryons**
- Read the axes and units first.

Conservation checks

- Quark charges: $u = +\frac{2}{3}e$, $d = -\frac{1}{3}e$, so $uud = +e$ and $udd = 0$.
- $n \to p + e^- + \overline{\nu}_e$ balances charge, baryon number and electron-lepton number.
- A free proton cannot decay to a neutron: the neutron is heavier.

The Hall effect

- R_H is negative for electrons and positive for holes. Conductivity alone cannot tell them apart.
- Fewer carriers give a larger Hall voltage, so semiconductors show the effect far more strongly than metals.
- With both electrons and holes present, a model weighted by their mobilities is needed.

$$V_H = \frac{IB}{nqt}, \qquad R_H = \frac{1}{nq}$$

Worked example: deuterium–tritium fusion

- Initial mass: 5.030151 u. Final mass: 5.011268 u.
- $Q = 0.018883 \times 931.5 = 17.6$ MeV.
- Momentum sharing gives most of this (about 14.1 MeV) to the light neutron.

Worked example: the Hall voltage in copper

- $V_H = \dfrac{IB}{net} = \dfrac{(5.0)(1.0)}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-4})} = 3.7 \times 10^{-6}$ V.
- Only a few microvolts, because copper has so many carriers.
- A semiconductor with 10^{22} carriers per m^3 would give a voltage about ten million times larger.

Common errors

- Judging stability by total binding energy instead of B/A.
- Calling the muon a meson.
- Forgetting the antineutrino in beta decay.

Common errors (2)

- Reading the sign of a Hall voltage without stating the current, field and lead directions.
- Expecting conductivity to reveal the sign of the carriers.

Key terms

- **Hall effect**: A sideways voltage across a conductor carrying current in a magnetic field.
- **binding energy per nucleon**: The binding energy divided by the number of nucleons; it measures stability.
- **Q value**: The energy released by a reaction, from the mass difference.
- **lepton**: A particle with no **quarks**, such as the electron, muon or a neutrino.

Check yourself

- A nucleus with $A = 10$ has $B = 80$ MeV. What is its **binding energy per nucleon**?
- What is the charge of a *uuu* baryon?
- A Hall coefficient is positive. What are the carriers?