Relativistic lifetime, energy and Doppler shift

GRE undergraduate preparation · local supported objectives

Relativistic lifetime, energy and Doppler shift

- Use **proper time** 固有时, **time dilation** and **length contraction** 长度收缩.
- Find a rest mass from energy and momentum with the **invariant** 不变量 $E^2 - (pc)^2$.
- Compute relativistic kinetic energy.
- Use the relativistic Doppler shift and velocity addition.

Proper time and dilation

- A clock moving with a particle measures its **proper time** τ_0; the lab measures $\gamma\tau_0$.
- Seen from the muon, the mountain is contracted by γ instead: the two views agree on how many muons survive.
- Decay is random, so these are mean lifetimes and mean distances.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \beta = \frac{v}{c}$$

The invariant mass

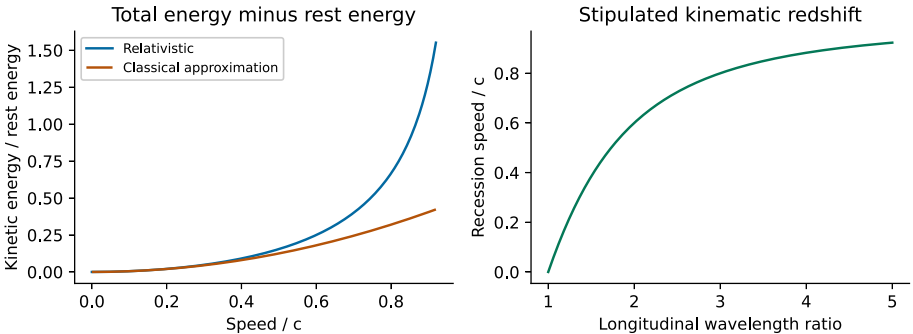
- So $mc^2 = \sqrt{E^2 - (pc)^2}$, never $E - pc$.
- For a system of particles, use the total energy and total momentum: this gives the **invariant mass 不变质量** of the system.
- Massless particles have $E = pc$.

$$E^2 - (pc)^2 = (mc^2)^2$$

Kinetic energy

- Work raises γ without limit, so a massive particle never reaches c.
- $K = (\gamma - 1)mc^2$.

Figure: Kinetic energy



- Classical $\frac{1}{2}mv^2$ underestimates kinetic energy at high speed; a wavelength ratio of 2 means 0.6c
- Read the axes and units first.

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Relativistic Doppler shift

- A longer observed wavelength is a **redshift** 红移; an approaching source gives $r < 1$.
- The low-speed rule $\beta \approx r - 1$ works only for small shifts.
- Cosmological redshifts of distant galaxies need a different model, based on the expansion of space.

$$r = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1+\beta}{1-\beta}}, \qquad \beta = \frac{r^2 - 1}{r^2 + 1}$$

Velocity addition

- The result never exceeds c. Light moves at c in every frame.
- Speeds do not simply add.

$$u' = \frac{u + v}{1 + uv/c^2}$$

Worked example: a fast particle's range

- $\gamma = 1/\sqrt{1-0.64} = 5/3$, so its lab lifetime is 5.0 μs.
- Mean distance: $0.80 \times (3.0 \times 10^{8}) \times (5.0 \times 10^{-6}) = 1200$ m.
- Without dilation: $0.80 \times (3.0 \times 10^{8}) \times (3.0 \times 10^{-6}) = 720$ m.

Worked example: masses and Doppler

- A particle has $E = 13$ GeV and $pc = 12$ GeV: $mc^2 = \sqrt{169 - 144} = 5$ GeV, not 1 GeV.
- Two 1.0 GeV photons moving in opposite directions: total momentum zero, so the system's **invariant mass** is 2.0 GeV/c^2. Moving the same way, it is zero.
- A line arrives at twice its emitted wavelength: $\beta = (4 - 1)/(4 + 1) = 0.6$.

Common errors

- Taking the rest energy as $E - pc$.
- Using $\frac{1}{2}mv^2$ at high speed.
- Applying dilation the wrong way: the proper lifetime is the shortest.

Common errors (2)

- Adding the masses of particles instead of finding the system's **invariant mass**.
- Adding velocities simply, giving speeds above c.

Key terms

- **time dilation**: A moving clock runs slow by the factor γ.
- **proper time**: The time measured by a clock moving with the object.
- **length contraction**: A moving object is shorter along its motion by the factor γ.
- **invariant**: A quantity with the same value in every inertial frame.

Check yourself

- A particle has $E = 5$ GeV and $pc = 4$ GeV. What is mc^2?
- A spectral line arrives at three times its emitted wavelength. What is β?
- A rocket at $0.6c$ fires a probe forward at $0.6c$ relative to itself. How fast is the probe in the lab?