

Relativistic lifetime, energy and Doppler shift

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A fast unstable particle can travel farther in the laboratory than its rest-frame lifetime multiplied by its speed would suggest. Relate proper lifetime to laboratory time and distance Use invariant energy–momentum and relativistic kinetic energy Infer longitudinal recession speed from wavelength ratio

Choose the system and model

Reasoning

For relative speed v , define $\beta=v/c$ and $\gamma=1/\sqrt{1-\beta^2}$. Proper time is measured along the particle's worldline by a clock at rest with it. If its proper mean lifetime is τ_0 , its laboratory mean lifetime is $\gamma\tau_0$ and mean travel distance is $v\gamma\tau_0$ at constant speed. The decay is statistical: mean lifetime is not a guaranteed decay time for each particle. At $\beta=0.8$, $\gamma=5/3$. Compute spacetime events consistently in one frame; proper time and coordinate time are different quantities.

Use the governing relation

Reasoning

Total energy is $E = \gamma mc^2$, momentum $p = \gamma mv$ and invariant $E^2 - p^2 c^2 = m^2 c^4$. When E and pc are expressed in the same energy units, find $mc^2 = \sqrt{E^2 - (pc)^2}$, not $E - pc$. The nonnegative square root is required for positive rest mass. For $E = 13$ GeV and $pc = 12$ GeV, rest energy is 5 GeV and mass is 5 GeV/ c^2 . A massless particle has $E = pc$ but need not have zero energy or momentum. These relations concern isolated-particle four-momentum, not classical mv at relativistic speed.

Apply the conditions

Reasoning

Work accelerating a particle from rest equals kinetic energy $K = E - mc^2 = (\gamma - 1)mc^2$. At $\beta = 0.6$, $\gamma = 1.25$ and $K = 0.25mc^2$, while the classical $\frac{1}{2}mv^2$ gives $0.18mc^2$. Classical kinetic energy is the low-speed expansion and becomes inaccurate near c . Finite acceleration work increases γ rather than allowing a massive particle to reach or exceed c . Keep total energy, rest energy and kinetic energy distinct when interpreting answer units.

Check the conclusion

Reasoning

For purely longitudinal relative recession in special relativity, wavelength ratio $r = \lambda_{\text{observed}} / \lambda_{\text{emitted}} = \sqrt{(1+\beta)/(1-\beta)}$. Rearranging gives $\beta = (r^2 - 1)/(r^2 + 1)$. A ratio $r=2$ gives $\beta=3/5$, not c times $r-1$ from a low-speed approximation. Blueshift uses $r < 1$ and a negative recession parameter under this convention. The formula assumes the shift is entirely kinematic; cosmological expansion or gravitational redshift requires a different model. State the question's stipulated model before interpreting a spectral ratio.

Worked reasoning 1

Reasoning

Total energy and momentum determine invariant rest energy.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$E^2 - (pc)^2 = (mc^2)^2, \quad mc^2 = \sqrt{E^2 - (pc)^2}.$$

Worked reasoning 3

Reasoning

For $E = 17 \text{ GeV}$ and $pc = 15 \text{ GeV}$,

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$mc^2 = \sqrt{(17 \text{ GeV})^2 - (15 \text{ GeV})^2} = 8 \text{ GeV}.$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

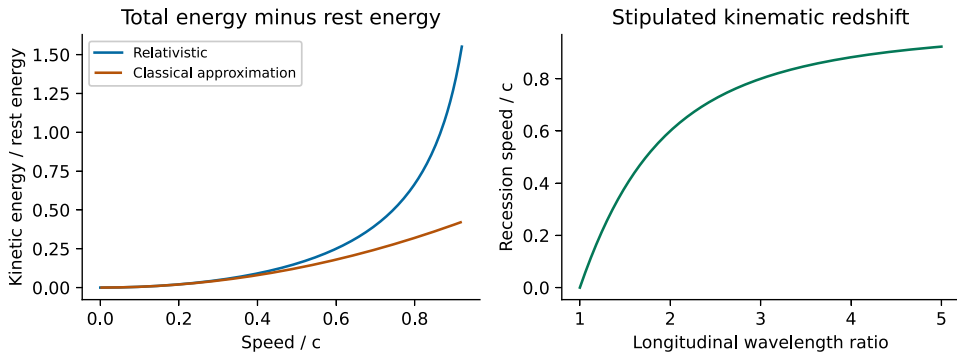
$$K = E - mc^2 = 17 \text{ GeV} - 8 \text{ GeV} = 9 \text{ GeV}.$$

Worked reasoning 7

Reasoning

Rest mass is $8 \text{ GeV}/c^2$. Do not subtract momentum directly from energy.

Read the original diagram



- Original GRE teaching diagram: Relativistic lifetime, energy and Doppler shift
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Equal-energy photons of energy E collide with directions separated by angle θ .
Derive the invariant mass of the pair. Check parallel and head-on limits. **[4]**

Independent transfer · reasoning

Reasoning

Total energy is $2E$; total momentum magnitude is $2(E/c)|\cos(\theta/2)|$.

$M^2 c^4 = 4E^2 - 4E^2 \cos^2(\theta/2) = 4E^2 \sin^2(\theta/2)$. Thus $Mc^2 = 2E \sin(\theta/2)$ for $0 \leq \theta \leq \pi$.

Parallel photons have zero pair invariant mass; head-on photons have $Mc^2 = 2E$.

Award vector momentum, invariant, result and limits. Each photon remains massless.

Check assumptions and limits

Reasoning

Do not use the proper lifetime as laboratory time, subtract pc from E to find rest mass, or apply the classical Doppler approximation to a large wavelength ratio.

Keep the technical distinctions

Reasoning

proper time : Time recorded by a clock moving with the object along its worldline. rest energy : Energy mc^2 associated with an object's rest mass.

Check yourself

- Define proper time and give its condition of use.
- Define rest energy and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.