

Counting statistics, uncertainty and dimensional models

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A detector with ten percent efficiency does not register exactly ten photons out of every hundred. The count fluctuates even with stable efficiency. Use binomial and Poisson count means and fluctuations Propagate small uncertainties with stated correlation assumptions Solve dimensional exponent constraints and check a model's units

Choose the system and model

Reasoning

For N independent incident particles each detected with probability p , count X is binomial: mean Np and variance $Np(1-p)$. The standard deviation is $\sqrt{Np(1-p)}$, not the variance itself. With large N and small p , a Poisson approximation has mean $\lambda=Np$ and variance λ , so deviation $\sqrt{\lambda}$. For $N=200$ and $p=0.05$, the exact mean is 10 and deviation $\sqrt{9.5} \approx 3.08$, while Poisson gives $\sqrt{10} \approx 3.16$. Neither model promises a fixed count. Correlated detections or dead time can invalidate independence.

Use the governing relation

Reasoning

For a smooth measured function $y(x_1, \dots)$, linearise changes using its partial derivatives. Independent small standard uncertainties combine in quadrature:

$\sigma_y^2 \approx \sum (\partial y / \partial x_i)^2 \sigma_i^2$. Correlated inputs require covariance cross terms. For $y = x^a$, fractional standard uncertainty is approximately $|a| \sigma_x / |x|$. Thus kinetic energy $K = \frac{1}{2}mv^2$ with negligible mass uncertainty has fractional uncertainty twice that of speed. With independent mass uncertainty, combine $(\sigma_m/m)^2 + (2\sigma_v/v)^2$. These approximations need small errors and a suitable local linear model.

Apply the conditions

Reasoning

Random scatter measures precision; a common calibration bias affects accuracy and does not disappear by averaging repeats. For independent repeated readings, the standard uncertainty of the mean falls as $1/\sqrt{n}$, but shared systematic error does not. The uncertainty of a physical spread and uncertainty of its estimated mean are different quantities. State whether a quoted percentage is a standard uncertainty, confidence interval or worst-case bound. Summing absolute contributions is a conservative bound, not the independent-standard-error quadrature rule.

Check the conclusion

Reasoning

Dimensional analysis equates powers of mass, length and time, rather than numerical sizes. For a Planck-length form $G^a \hbar^b c^d$, dimensions are $[G]=L^3/(MT^2)$, $[\hbar]=ML^2/T$ and $[c]=L/T$. Requiring length gives $-a+b=0$, $3a+2b+d=1$ and $-2a-b-d=0$. Hence $a=b=1/2$ and $d=-3/2$, so length is $\sqrt{G\hbar/c^3}$. Dimensional analysis cannot determine an arbitrary dimensionless coefficient or prove that the chosen constants are physically sufficient. Reject a formula with wrong units before inserting numbers.

Worked reasoning 1

Reasoning

For independent small standard uncertainties differentiate first.

in $K = mv^2/2$,

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

Worked reasoning 3

Reasoning

With relative uncertainties 3% in mass and 2% in speed,

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

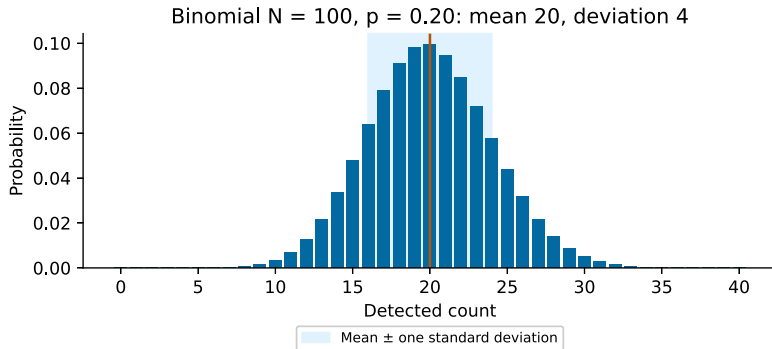
$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

Worked reasoning 5

Reasoning

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.

Read the original diagram



- Original GRE teaching diagram: Counting statistics, uncertainty and dimensional models
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A sample interval contains 900 counts; an independent equal-duration background interval contains 100. Estimate net counts and their standard uncertainty. If both intervals are doubled at stable rates, what happens to relative uncertainty? **[4]**

Independent transfer · reasoning

Reasoning

Net count is 800. Independent Poisson variances add despite subtraction:

$\sigma = \sqrt{900 + 100} = \sqrt{1000} = 31.62$ counts, relative 0.03953. Doubled intervals give net 1600 and variance 2000, so relative uncertainty is divided by $\sqrt{2}$. Award net, variance addition, uncertainty and time scaling. This assumes stable rates and no dead-time correlations.

Check assumptions and limits

Reasoning

Do not confuse mean count with guaranteed count, variance with standard deviation, or a random standard error with a calibration bias. Dimensional consistency is necessary but not sufficient.

Keep the technical distinctions

Reasoning

standard uncertainty : Uncertainty expressed as a standard deviation under a stated measurement model. Poisson approximation : Rare independent-event count model with variance equal to its mean.

Check yourself

- Define standard uncertainty and give its condition of use.
- Define Poisson approximation and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.