Gas processes, entropy and reversible cycles

GRE undergraduate preparation · local supported objectives

Gas processes, entropy and reversible cycles

- Apply the first law along different paths, with clear signs.
- Read work and heat from P–V and T–S diagrams.
- Treat entropy as a **state function** 状态函数.
- Use the Carnot limits for engines, refrigerators and heat pumps.

The first law along a path

- Constant volume: $W = 0$, so $\Delta U = Q$.
- **Isothermal** 等温的: $\Delta U = 0$, so $Q = W$.
- **Adiabatic** 绝热的: $Q = 0$, so $\Delta U = -W$. The temperature changes.
- Round a complete **cycle** 循环: $\Delta U = 0$, so the net heat in equals the net work out.

Reading P–V diagrams

- An isotherm is $PV = $ constant; an adiabat, $PV^{\gamma} = $ constant, is steeper.
- A cycle run clockwise does positive net work, equal to the enclosed area.
- A rectangle from 1 to 3×10^{5} Pa and 0.001 to 0.004 m^{3} encloses $2 \times 10^{5} \times 0.003 = 600$ J.

Adiabatic processes

- $\gamma = 5/3$ for a monatomic gas and $7/5$ for air (diatomic) near room temperature.
- A diesel engine ignites its fuel by adiabatic compression alone.

$$PV^{\gamma} = \text{constant}, \qquad TV^{\gamma-1} = \text{constant}$$

Entropy

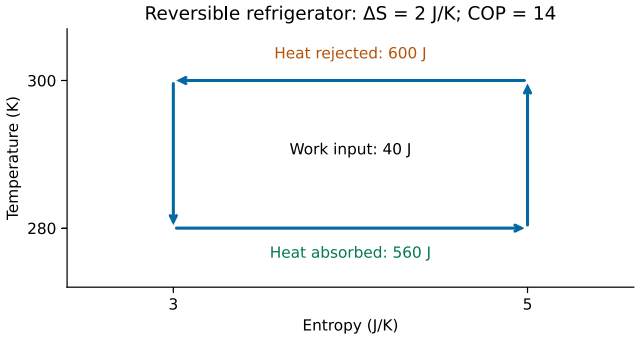
- Reversible **isothermal** expansion: $\Delta S_{\text{gas}} = nR\ln(V_2/V_1)$; the reservoir loses the same amount.
- A reversible process has zero total entropy change, though the system's entropy can change.
- On a T–S diagram, the area under a reversible path is the heat taken in.

Engines, refrigerators and heat pumps

- For the same temperatures, $\mathrm{COP}_{\mathrm{pump}} = \mathrm{COP}_{\mathrm{fridge}} + 1$, because the heat delivered includes the work.
- Both COPs grow as the temperatures get closer.

$$\eta = 1 - \frac{T_C}{T_H}, \qquad \mathrm{COP}_{\mathrm{fridge}} = \frac{T_C}{T_H - T_C}, \qquad \mathrm{COP}_{\mathrm{pump}} = \frac{T_H}{T_H - T_C}$$

Figure: Engines, refrigerators and heat pumps



- For a reversible refrigerator, COP $= 560/40 = 14$
- Read the axes and units first.

Worked example: a Carnot refrigerator

- Heat taken from inside: $Q_C = T_C \Delta S = 280 \times 2.0 = 560$ J. Heat rejected: $Q_H = 300 \times 2.0 = 600$ J.
- Work: $W = 600 - 560 = 40$ J, so COP $= 560/40 = 14 = 280/20$.
- As a **heat pump** between the same temperatures, it would deliver 600 J for 40 J of work: COP 15.

Worked example: compressing air

- $T_2 = T_1 (V_1/V_2)^{\gamma - 1} = 300 \times 2^{0.4} = 396$ K.
- $P_2/P_1 = 2^{1.4} = 2.64$, more than the factor 2 an **isothermal** compression would give.
- The work done on the gas becomes internal energy, which is why a bicycle pump gets warm.

Common errors

- Thinking "adiabatic" means constant temperature.
- Using litres and kilopascals in $W = P\Delta V$ without converting.
- Calling a COP of 14 an efficiency of 1400%.

Common errors (2)

- Saying a reversible process leaves the system's entropy unchanged; only the total is unchanged.
- Using degrees Celsius in the Carnot formulas.

Key terms

- **heat pump**: A device that uses work to move heat from a cold place to a warm one, in order to heat the warm one.
- **coefficient of performance**: Useful heat moved divided by the work supplied.
- **state function**: A quantity that depends only on the present state, not on how it was reached.
- **isothermal**: At constant temperature.

Check yourself

- A rectangular cycle runs between 1×10^5 and 2×10^5 Pa and between 2 and 5 litres. What is the net work?
- In an adiabatic expansion, does the temperature of an ideal gas change?
- What is the largest COP of a **heat pump** between 270 K and 300 K?