

Gas processes, entropy and reversible cycles

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A refrigerator can move more heat than the work supplied to it. That ratio is a coefficient of performance, not an efficiency that must be below one. Apply first-law work and heat signs to gas processes Compare reversible entropy balances and P–V cycle areas Distinguish engine efficiency from refrigerator coefficient of performance

Choose the system and model

Reasoning

Use $\Delta U = Q - W$, where Q is heat entering the system and W is work done by it. Quasistatic boundary work is $\int P \, dV$, positive during expansion. For a fixed amount of ideal gas, internal energy depends only on temperature. At constant volume $W=0$, so $\Delta U=Q$; for an isothermal ideal-gas process $\Delta U=0$, so $Q=W$. Adiabatic means $Q=0$, not constant temperature. Over a cycle, the state returns and $\Delta U=0$, so net Q equals net W even though individual legs have different heat transfers.

Use the governing relation

Reasoning

For reversible isothermal expansion of an ideal gas, PV is constant. With constant heat capacities, a reversible adiabatic path obeys $PV^\gamma = \text{constant}$, with $\gamma = C_p/C_v > 1$; through the same initial state its pressure falls faster as volume increases. An isobaric path is horizontal on a P – V plot. A clockwise closed cycle has positive $\oint P \, dV$; reversing direction changes the sign. For a rectangular loop the magnitude is $\Delta P \Delta V$, and a triangular loop has half the corresponding bounding-rectangle area. Use Pa and m^3 for joules, not an unconverted litre value.

Apply the conditions

Reasoning

For reversible heat transfer, $dS = \delta Q_{\text{rev}}/T$; entropy is a state function while heat is path dependent. A reversible system-plus-environment process has zero total entropy change, but the system's entropy alone can increase or decrease. Irreversible spontaneous processes produce nonnegative total entropy. A reversible isothermal expansion increases the gas entropy by $nR \ln(V_2/V_1)$; the reservoir loses the same amount. Reversibility does not require constant system temperature, zero work or zero internal-energy change.

Check the conclusion

Reasoning

A reversible engine between T_H and T_C has efficiency $W/Q_H = 1 - T_C/T_H$. A reversible refrigerator instead has $\text{COP} = Q_C/W = T_C/(T_H - T_C)$, and a heat pump has $Q_H/W = T_H/(T_H - T_C)$. Use absolute kelvin temperatures. On a reversible T - S diagram, heat magnitude along an isotherm is T times the entropy change; the Carnot rectangle's area is the work magnitude. Refrigeration moves heat from cold to hot by consuming work, reversing the engine cycle. COP can exceed one without violating conservation because the moved heat is not supplied solely by the work.

Worked reasoning 1

Reasoning

A reversible refrigerator transfers entropy ΔS from T_c to T_h .

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$Q_c = T_c \Delta S = (270 \text{ K})(3.0 \text{ J/K}) = 810 \text{ J}.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$Q_h = T_h \Delta S = (300 \text{ K})(3.0 \text{ J/K}) = 900 \text{ J}.$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

$$W_{in} = Q_h - Q_c = 900 \text{ J} - 810 \text{ J} = 90 \text{ J}.$$

Worked relation 8

Reasoning

Apply this relation only under the conditions just stated.

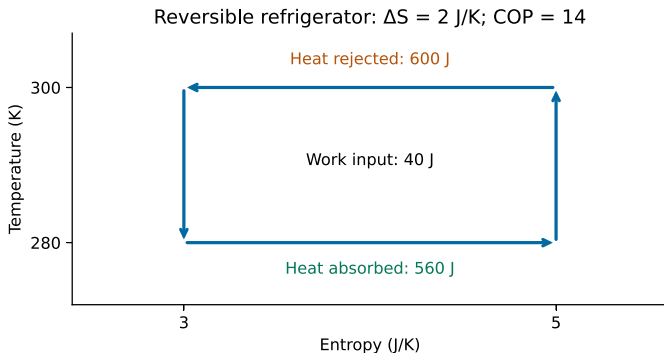
$$\text{COP} = Q_c/W_{in} = (810 \text{ J})/(90 \text{ J}) = 9.$$

Worked reasoning 9

Reasoning

COP above one is possible because heat is moved, rather than all produced from work.

Read the original diagram



- Original GRE teaching diagram: Gas processes, entropy and reversible cycles
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A refrigerator removes 600 J from 280 K and rejects 650 J to 300 K per cycle. Check energy balance and total entropy change. Is the proposed cycle possible? Find the minimum work needed for the same cooling between these reservoirs. ****[5]****

Independent transfer · reasoning

Reasoning

Energy requires 50 J work input. Reservoir entropy change is $650/300 - 600/280 = +0.02381 \text{ J/K} > 0$, so the complete cycle does not violate the second law. Reversible minimum work is $Q_c(T_h/T_c - 1) = 600(300/280 - 1) = 42.86 \text{ J}$. The proposed cycle is possible thermodynamically and uses more than minimum work. Award work, correct entropy arithmetic, sign/possibility, reversible bound and comparison.

Check assumptions and limits

Reasoning

Zero total entropy production does not mean zero system entropy change. A refrigerator uses $TC/(T_H - T_C)$, not the engine efficiency formula.

Keep the technical distinctions

Reasoning

entropy production : Nonnegative total entropy generated by irreversibility.
coefficient of performance : Useful heat transferred divided by work input for
a refrigerator or heat pump.

Check yourself

- Define entropy production and give its condition of use.
- Define coefficient of performance and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.