

Optical instruments, path differences and refraction

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A gas cell in one arm of a Michelson interferometer is crossed twice. Counting only one passage halves the inferred refractive-index change. Calculate telescope magnification and grating resolving power Relate Michelson fringe counts to double-pass optical path changes Trace surface normals before applying Snell and reflection laws

Choose the system and model

Reasoning

For an ideal astronomical telescope adjusted for relaxed viewing at infinity, the objective forms its focal-plane image and the eyepiece collimates the emerging light. Lens separation is $f_{\text{objective}} + f_{\text{eyepiece}}$; angular magnification magnitude is $f_{\text{objective}}/f_{\text{eyepiece}}$. The usual two-converging-lens telescope produces an inverted image, represented by a negative signed angular magnification under a consistent convention. A finite final-image distance changes lens separation; do not use the infinity adjustment without checking the condition.

Use the governing relation

Reasoning

Resolving power $R = \lambda / \Delta\lambda$ describes the smallest distinguishable wavelength separation near λ . For an ideal diffraction grating in order m with N illuminated slits, $R = mN$. This is a resolution criterion, not simply the angular position formula $d \sin\theta = m\lambda$. Increasing illuminated slit count sharpens the principal peaks; increasing slit spacing mainly changes their angular locations. A measured $\lambda = 600 \text{ nm}$ and $\Delta\lambda = 3 \text{ nm}$ gives $R = 200$, independent of converting both lengths to metres because their ratio uses matching units.

Apply the conditions

Reasoning

Michelson interference depends on the optical path difference between the arms. If one arm contains a gas cell of geometric length L and index changes from n to 1 upon evacuation, a double passage changes path by $2L(n-1)$. If N fringes pass, the magnitude of this change is $N\lambda$, so $n-1=N\lambda/(2L)$. For a moving mirror instead, displacement Δx gives path change $2\Delta x$. Fringes count phase cycles; one fringe is one wavelength of optical path, not one wavelength of mirror motion.

Check the conclusion

Reasoning

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ uses angles from the local surface normal. Reflection has equal incoming and outgoing angles from that same normal. In a rectangular block, normals of neighbouring faces are perpendicular: an incidence angle measured from one normal is complementary to the angle of that same ray from the other. Draw the ray and normals before substitution. For incidence from index n into air, total internal reflection occurs only when $\sin\theta > 1/n$; equality is the critical grazing case. A stated partly refracted ray must satisfy the transmission condition.

Worked reasoning 1

Reasoning

A Michelson interferometer sends light twice through a gas cell of length L . Evacuation changes path by $2L(n - 1)$. N fringes correspond to path change $N\lambda$.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$n - 1 = \frac{N\lambda}{2L} = \frac{(120)(500 \times 10^{-9} \text{ m})}{2(0.10 \text{ m})} = 3.0 \times 10^{-4}.$$

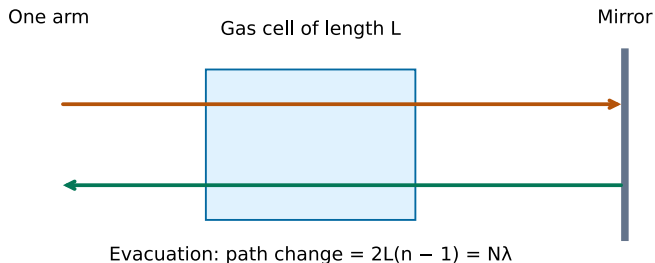
Worked reasoning 3

Reasoning

Thus $n = 1.00030$. Moving a mirror is also double-pass; a fringe is not one wavelength of mirror motion.

Read the original diagram

Two passages through the same cell



- Original GRE teaching diagram: Optical instruments, path differences and refraction
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A grating with spacing $d = 2.0$ micrometres illuminates $N = 5000$ slits at wavelength 600 nm. Find the largest physically allowed order and its ideal resolving limit in wavelength. State the different roles of spacing and illuminated slit count. **[4]**

Independent transfer · reasoning

Reasoning

$|m\lambda/d| \leq 1$ gives $m \leq 3.33$, hence largest integer order 3. $R = mN = 15000$ and $\Delta\lambda = \lambda/R = 600 \text{ nm}/15000 = 0.040 \text{ nm}$. Spacing sets the order angles; illuminated slit count narrows the peaks and improves resolution. Award order condition, order, resolving limit and distinction. The ideal limit assumes uniform illumination.

Check assumptions and limits

Reasoning

Use optical path, not just geometric distance. Refraction angles are measured from the normal of the actual surface, and telescope focal-length addition assumes a final image at infinity.

Keep the technical distinctions

Reasoning

resolving power : Wavelength divided by the smallest resolvable wavelength
separation. optical path length : Geometric path weighted by refractive index
along the ray.

Check yourself

- Define resolving power and give its condition of use.
- Define optical path length and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.