Acoustic Doppler echoes and standing-wave boundaries

GRE undergraduate preparation · local supported objectives

Acoustic Doppler shifts and standing waves

- Use the different formulas for a moving source and a moving observer.
- Handle echoes, which are shifted twice.
- Find standing-wave frequencies on strings and in pipes, with **nodes** 波节 and **antinodes** 波腹.
- Use **beats** 拍 to compare two frequencies.

Moving source or moving observer

- Source approaching a still observer at u_s: $f' = f\dfrac{c}{c - u_s}$.
- Observer approaching a still source at u_o: $f' = f\dfrac{c + u_o}{c}$.
- Receding changes the sign of the speed. The two formulas are not interchangeable: they give different shifts for the same speed.

Echoes from a fixed wall

- For small u/c, the fractional shift is about $2u/c$. Police radar and ultrasound blood-flow meters use this.
- An echo is shifted twice.

$$f_{\text{echo}} = f\frac{c+u}{c-u}$$

Standing waves on strings

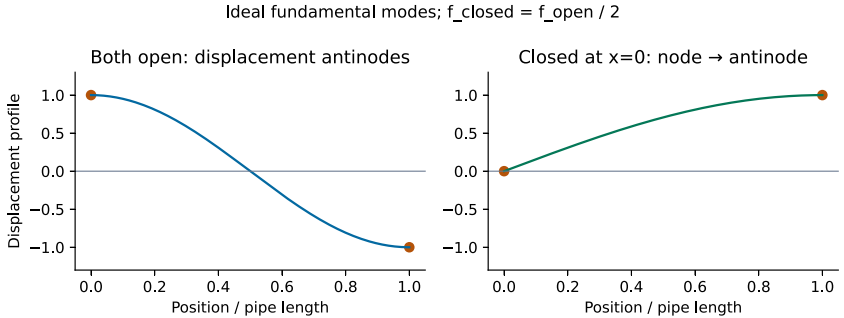
- The lowest frequency is the **fundamental 基频**; the others are **harmonics 谐波**.
- Four times the tension doubles the wave speed and every frequency.

$$f_n = \frac{nv}{2L}, \qquad v = \sqrt{T/\mu}$$

Pipes: nodes and antinodes

- Both ends open: $f_n = \dfrac{nc}{2L}$, all whole numbers n.
- One end closed: $f_n = \dfrac{(2n-1)c}{4L}$, odd multiples of the fundamental only.

Figure: Pipes: nodes and antinodes



- Identify the axes, units, directions and the stated model.

Beats

- Musicians tune by removing **beats** against a reference note.
- Two notes of nearly equal frequency add to a sound whose loudness rises and falls.

$$f_{\text{beat}} = |f_1 - f_2|$$

Worked example: a siren and its echo

- Frequency reaching the wall: $500 \times 340/330 = 515$ Hz.
- Echo heard by the driver: $500 \times 350/330 = 530$ Hz.
- The driver hears the siren (500 Hz) and the echo (530 Hz) together: 30 **beats** per second, close to the estimate $2u/c \times 500 = 29$ Hz.

Worked example: closing one end of a pipe

- Both ends open: $f_n = n(340)/(2 \times 0.85) = 200, 400, 600$ Hz, ...
- One end closed: $f_n = (2n-1)(340)/(4 \times 0.85) = 100, 300, 500$ Hz, ...
- The fundamental halves, and none of the old frequencies survives: they were all even multiples of $c/4L$.

Common errors

- Using the source formula when the observer moves, or the reverse.
- Measuring speeds relative to the ground when the air is moving (wind).
- Shifting an echo only once.

Common errors (2)

- Allowing even **harmonics** in a pipe closed at one end.
- Putting a node at an open end.

Key terms

- **Doppler effect**: The change in observed frequency caused by motion of the source or observer.
- **node**: A point of a standing wave that does not move.
- **antinode**: A point of a standing wave with the largest motion.
- **beats**: The regular rise and fall in loudness of two notes with nearly equal frequencies.

Check yourself

- A 680 Hz source moves away from you at 20 m/s; $c = 340$ m/s. What do you hear?
- A string 0.65 m long carries waves at 260 m/s. What is its fundamental?
- Two tuning forks of 440 Hz and 444 Hz sound together. What is the beat frequency?