Circuit power, induction and charged-particle motion

GRE undergraduate preparation · local supported objectives

Circuit power, induction and charged-particle motion

- Reduce resistor networks and find each branch's voltage and current.
- Use rms values, **impedance 阻抗** and **resonance** in AC circuits.
- Apply Faraday's law to coils.
- Follow charges in magnetic fields: radii, cyclotron frequency, the **velocity selector 速度选择器**.

Reduce, then expand

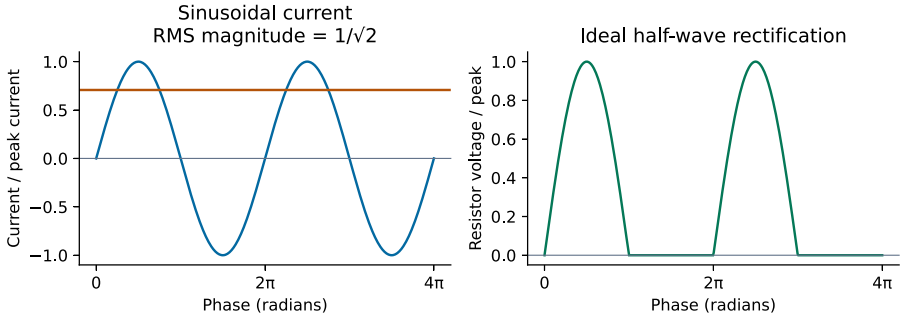
- Kirchhoff's current rule: the currents into a node sum to zero.
- Kirchhoff's voltage rule: the voltage changes round any loop sum to zero.
- Not every resistor "sees" the battery voltage; series resistors share it.

AC power

- Ideal inductors and capacitors take no average power.
- Series RLC: $|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. At **resonance 共振**, $\omega_0 = 1/\sqrt{LC}$, $|Z| = R$ and the current is largest.
- A single ideal **diode 二极管** passes only half of each cycle: half-wave rectification.

$$P = I_{\text{rms}}^2 R = V_{\text{rms}} I_{\text{rms}} \cos\varphi$$

Figure: AC power



- Identify the axes, units, directions and the stated model.

Faraday's law in a coil

- The induced current opposes the change in flux, not the field itself.
- A coil spinning at ω in a steady field gives $\mathcal{E} = NBA\omega \sin \omega t$: a generator.

$$\mathcal{E} = -N\frac{d\Phi}{dt}$$

Magnetic fields of currents

- Long straight wire: $B = \dfrac{\mu_0 I}{2\pi r}$.
- Centre of a circular loop: $B = \dfrac{\mu_0 I}{2R}$, times N for a flat coil.
- Long solenoid: $B = \mu_0 n I$ inside, with n turns per metre, and almost zero outside.

Charges in magnetic fields

- ω_c does not depend on speed. Deuterons and alpha particles both have half the proton's ω_c, since q/m is halved.
- **Velocity selector**: crossed $\mathbf{E}$ and $\mathbf{B}$ let through undeflected only charges with $qE = qvB$, so $v = E/B$.

$$r = \frac{mv}{|q|B}, \qquad \omega_c = \frac{|q|B}{m}$$

Worked example: a resistor network

- $3 \parallel 6 = 2\ \Omega$, so the total is 4 Ω and the current is 6 A.
- The 2 Ω resistor takes 12 V; the parallel pair has 12 V, so the branch currents are 4 A and 2 A.
- Power: 72 W, 48 W and 24 W, totalling 144 W $= 24 \times 6$.

Worked example: a cyclotron

- Frequency: $f = \dfrac{qB}{2\pi m} = \dfrac{(1.6 \times 10^{-19})(1.5)}{2\pi(1.67 \times 10^{-27})} = 23$ MHz.
- At a radius of 0.50 m: $v = qBr/m = 7.2 \times 10^{7}$ m/s.
- Kinetic energy: $\frac{1}{2}mv^2 = 4.3 \times 10^{-12}$ J $= 27$ MeV. At $v = 0.24c$, relativity changes this by only a few per cent.

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Common errors

- Giving every resistor the full battery voltage.
- Halving the power again after using rms values.
- Measuring the flux angle from the plane of the coil instead of its normal.

Common errors (2)

- Thinking a magnetic field speeds up a charge; it only turns it.
- Thinking the cyclotron frequency grows with speed (it does not, until relativity matters).

Key terms

- **cyclotron**: An accelerator in which charges circle in a magnetic field and are pushed by an alternating voltage.
- **impedance**: The AC opposition to current, combining resistance and reactance.
- **velocity selector**: Crossed electric and magnetic fields that pass charges of one speed only.
- **rms**: Root mean square; for a sine wave, the peak value divided by $\sqrt{2}$.

Check yourself

- The mains voltage has a peak of 170 V. What is its rms value?
- A **velocity selector** has $E = 6000$ V/m and $B = 0.20$ T. Which speed passes?
- How does an alpha particle's cyclotron frequency compare with a proton's in the same field?