Electrostatic superposition, flux and conductors

GRE undergraduate preparation · local supported objectives

Electrostatic superposition, flux and conductors

- Add electric fields as vectors and potentials as numbers.
- Find the **flux** 通量 through open as well as closed surfaces.
- Use **electrostatic equilibrium** 静电平衡 in conductors and **induced charge** 感应电荷.
- Recognise the **method of images**.

Coulomb's law and superposition

- Resolve each field into components, add, then find the size. Forces of 3 N to the right and 4 N upwards combine to 5 N, not 7 N.
- Potential adds as a scalar: $V = \sum Q_i/(4\pi\varepsilon_0 r_i)$. This is easier, but $V = 0$ does not mean $\mathbf{E} = 0$.

$$\mathbf{E} = \sum_i \frac{Q_i(\mathbf{r} - \mathbf{r}_i)}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_i|^3}$$

Flux through open surfaces

- A point charge at the centre of a cube sends $q/(6\varepsilon_0)$ through each face.
- A point charge beside an infinite plane sends half its flux through the plane, $q/(2\varepsilon_0)$, whatever the distance.

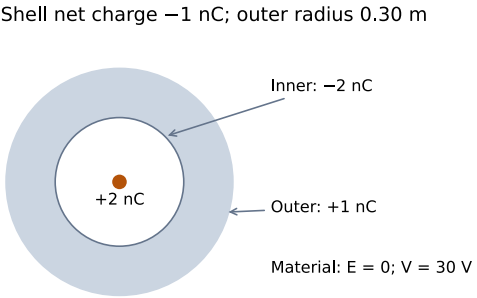
Conductors in equilibrium

- Any net charge lies on the surfaces.
- The potential of the metal is constant, but not usually zero.
- Just outside, the field is perpendicular to the surface.

A charge inside a shell

- The inner surface carries $-Q$; the outer surface carries $q + Q$.
- Cavity ($r < a$): $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$. Metal: $E = 0$. Outside ($r > b$): $E = \dfrac{Q+q}{4\pi\varepsilon_0 r^2}$.
- V is continuous everywhere; E jumps at each charged surface. An off-centre charge still induces $-Q$ inside, but the cavity field is then not radial.

Figure: A charge inside a shell



- Induced charges make E = 0 in the metal; the shell's potential is k(+1 nC)/0.30 m = 30 V
- Read the axes and units first.

exercises.lol exercises.com.cn

The method of images

- A charge q at height d above a grounded plane feels the force of $-q$ at distance $2d$: $F = \dfrac{q^2}{4\pi\varepsilon_0(2d)^2}$, towards the plane.
- The image is used only to find the field above the plane; there is no field inside the conductor.

Worked example: the shell at 30 V

- Inner surface: -2.0 nC. Outer surface: $-1.0-(-2.0)=+1.0$ nC.
- Potential of the metal: $V=\dfrac{(9.0\times10^{9})(1.0\times10^{-9})}{0.30}=30$ V.
- Field in the metal: zero, even though the potential is 30 V.

Worked example: two equal charges

- Each charge is 0.50 m away, giving $E = (9.0 \times 10^{9})(1.0 \times 10^{-9})/0.25 = 36$ N/C each.
- The x-components cancel; the y-components add: $2 \times 36 \times 0.80 = 58$ N/C upwards.
- $V = 2 \times (9.0)/0.50 = 36$ V.

Common errors

- Adding field magnitudes without resolving into components.
- Thinking that $E = 0$ inside a conductor means $V = 0$.
- Using Gauss's law on an open surface without a symmetry argument.

Common errors (2)

- Putting the induced $-Q$ on the outer surface.
- Forgetting that the image charge sits at $2d$ from the real one, not d.

Key terms

- **superposition**: The total field of several charges is the vector sum of their separate fields.
- **flux**: The amount of field passing through a surface.
- **electrostatic equilibrium**: The state in which no charges move in a conductor, so $E = 0$ inside it.
- **induced charge**: Charge that gathers on a conductor's surface because of nearby charges.

Check yourself

- Forces of 3.0 N and 4.0 N act at right angles on a charge. What is the net force?
- A point charge sits at the centre of a cube. What flux passes through one face?
- What is the field inside the metal of the 30 V shell, and what is the potential there?