

Electrostatic superposition, flux and conductors

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A charged conductor can have zero field inside its material while its potential remains nonzero. Add Coulomb forces as vectors using the actual geometry Compute flux from symmetry or solid angle with a stated orientation Find conductor fields and potentials using induced surface charges

Choose the system and model

Reasoning

For a point charge Q at position $\mathbf{r}_0$, $\mathbf{E}(\mathbf{r}) = Q(\mathbf{r} - \mathbf{r}_0) / (4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_0|^3)$. Superpose vectors, not field magnitudes. For two positive charges attracting an electron, forces in the same direction add; opposite directions subtract; perpendicular components combine by Pythagoras. Define the observation point and the direction of each force first. Potential is the scalar sum $V = \sum Q / (4\pi\epsilon_0 r)$ with zero at infinity. A zero potential does not generally mean a zero field, since $\mathbf{E} = -\nabla V$.

Use the governing relation

Reasoning

Gauss's law is the closed-surface integral $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$. It determines a local field simply only when symmetry makes the normal field uniform or makes other flux contributions zero. A charge near an infinite plane sends half its total flux through that plane in magnitude: the plane subtends solid angle 2π out of 4π . The result is independent of distance and lateral position, but its sign depends on the chosen plane normal and charge sign. An open plane does not enclose a charge; using closed-surface wording for it is incorrect.

Apply the conditions

Reasoning

In electrostatic equilibrium, the electric field inside conducting material is zero and the conductor has constant potential. Place a charge Q at the centre of a conducting spherical shell with inner radius a , outer radius b and shell net charge q . A Gaussian surface inside the material requires inner-surface charge $-Q$, so the outer surface has $q+Q$. Spherical symmetry makes the external field that of total $Q+q$ at the centre. The material's potential, zero at infinity, is $(Q+q)/(4\pi\epsilon_0 b)$, independent of the material observation radius.

Check the conclusion

Reasoning

In the cavity of that centred-charge shell, the field is $Q/(4\pi\epsilon_0 r^2)$ radially for $0 < r < a$. Potential includes both the point-charge contribution and constant shell contributions; continuity holds across each surface even though the normal field jumps at a surface charge. Keep cavity, conductor material and exterior separate. An off-centre charge still induces total inner charge $-Q$, but its cavity field is no longer the simple centred radial field. Zero interior conductor field follows equilibrium, not an assumption that every surface charge distribution is uniform.

Worked reasoning 1

Reasoning

A centred point charge Q lies in a conducting spherical shell of net charge q . Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{inner} = -Q$. Charge conservation gives $Q_{outer} = q + Q$. With outer radius b and zero potential at infinity,

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$V_{metal} = \frac{q + Q}{4\pi\epsilon_0 b} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-9} \text{ C})}{0.30 \text{ m}} = 30 \text{ V}.$$

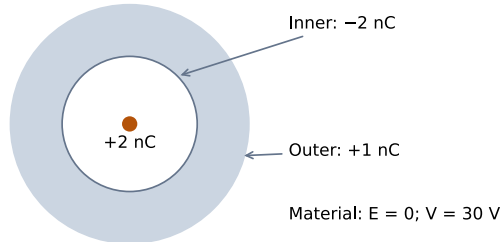
Worked reasoning 3

Reasoning

Here $Q = +2 \text{ nC}$ and $q = -1 \text{ nC}$. The cavity is not conducting material; its field need not vanish.

Read the original diagram

Shell net charge -1 nC ; outer radius 0.30 m



- Original GRE teaching diagram: Electrostatic superposition, flux and conductors
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Move Q away from the centre while keeping it inside the closed cavity. The shell's exterior remains a sphere, with no external charges. Which of inner charge total, inner charge distribution, metal potential and exterior field retain their centred values?

Explain using Gauss's law and electrostatic uniqueness. **[5]**

Independent transfer · reasoning

Reasoning

The inner total remains $-Q$ because the metal field stays zero, but its distribution is generally nonuniform. Outer total remains $q+Q$. The spherical exterior is equipotential; uniqueness with zero at infinity gives $V = k(q + Q)/r$ outside, so metal potential and exterior field retain their centred values. Cavity field changes. Award inner total, redistribution, outer total, uniqueness argument and unchanged exterior/potential.

Check assumptions and limits

Reasoning

Do not confuse the open-plane half-flux result with enclosed charge. A conductor's zero field fixes a constant potential, not necessarily zero potential.

Keep the technical distinctions

Reasoning

solid angle : Angular area subtended by a surface, measured in steradians.

induced surface charge : Charge rearranged on a conductor to satisfy electrostatic equilibrium.

Check yourself

- Define solid angle and give its condition of use.
- Define induced surface charge and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.