Coupled acceleration and relative motion

GRE undergraduate preparation · local supported objectives

Coupled acceleration and relative motion

- Use the **constraint** 约束 of a string to link accelerations.
- Write Newton's law for each body separately.
- Handle pulleys with mass.
- Change between **reference frames** 参考系 with **relative velocity** 相对速度.

One string, one acceleration

- Mass m_1 on a smooth table pulled by a hanging mass m_2: $T = m_1 a$ and $m_2 g - T = m_2 a$.
- So $a = \dfrac{m_2 g}{m_1 + m_2}$ and $T = \dfrac{m_1 m_2 g}{m_1 + m_2}$.
- T is less than $m_2 g$; if they were equal, m_2 could not accelerate.

The Atwood machine

- Nearly equal masses give a small acceleration, which is why Atwood used them.
- The tension lies between the two weights.

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}, \qquad T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

When the pulley is not ideal

- $(T_2 - T_1)R = I\alpha$ and $a = R\alpha$.
- The pulley adds I/R^2 to the "inertia" in the denominator: $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2 + I/R^2}$.

Relative velocity

- A payload released from an aircraft flying level at 80 m/s keeps that horizontal velocity. After 3 s its ground velocity is $(80, -30)$ m/s, but relative to the aircraft it is $(0, -30)$ m/s: seen from the aircraft, it drops straight down.
- A boat heading straight across a river at 4 m/s relative to water that flows at 3 m/s moves at 5 m/s over the ground.

Figure: Relative velocity

Subtract aircraft velocity; components in m/s

Velocity after 3 s	Horizontal	Vertical
Ground	80	−30
Aircraft	80	0
Payload relative to aircraft	0	−30

- Subtract the aircraft's velocity component by component
- Read the axes and units first.

Check the limits

- Table pulley: $m_1 \to 0$ gives $a \to g$, $T \to 0$; $m_1 \to \infty$ gives $a \to 0$, $T \to m_2 g$.
- Energy check without friction: $m_2 g s = \frac{1}{2}(m_1 + m_2)v^2$ agrees with $v^2 = 2as$.

Worked example: a table and a pulley

- $a = \dfrac{2.0 \times 10}{5.0} = 4.0$ m/s^2 and $T = 3.0 \times 4.0 = 12$ N, less than the 20 N weight.
- After falling 0.50 m from rest: $v^2 = 2(4.0)(0.50)$, so $v = 2.0$ m/s.
- Energy check: $2.0 \times 10 \times 0.50 = 10$ J $= \frac{1}{2}(5.0)(2.0)^2$.

Worked example: a pulley with mass

- The pulley adds $I/R^2 = \frac{1}{2}(2.0) = 1.0$ kg.
- $a = \dfrac{(3.0 - 2.0)(10)}{2.0 + 3.0 + 1.0} = 1.7$ m/s^2, compared with 2.0 m/s^2 for an ideal pulley.
- The tensions now differ: $T_1 = 2.0(10 + 1.7) = 23$ N and $T_2 = 3.0(10 - 1.7) = 25$ N.

Common errors

- Setting the tension equal to the hanging weight.
- Using one equation for the whole system and then trying to find the tension from it.
- Assuming equal tensions on both sides of a pulley that has mass.

Common errors (2)

- Adding frame velocities instead of subtracting them.
- Using the energy check when friction acts.

Key terms

- **pulley**: A wheel that changes the direction of a string's pull.
- **tension**: The pulling force a string or rope exerts.
- **constraint**: A condition that links the motions of bodies, such as a fixed string length.
- **reference frame**: The viewpoint and coordinates from which motion is measured.

Check yourself

- An Atwood machine has masses of 4.0 kg and 6.0 kg; $g = 10$ m/s^2. Find a and T.
- Rain falls vertically at 8.0 m/s. A car drives at 6.0 m/s. How fast does the rain hit the windscreen, as seen from the car?
- In the table pulley, what happens to T when m_1 becomes very large?