Rotation, buoyancy and terminal-speed balances

GRE undergraduate preparation · local supported objectives

Rotation, buoyancy and terminal speed

- Use **torque 力矩** and **moment of inertia 转动惯量** about the right axis.
- Share kinetic energy between translation and rotation in rolling.
- Apply Archimedes' principle to floating and composite objects.
- Find **terminal speed 终端速度** from the drag law.

Torque about a pivot

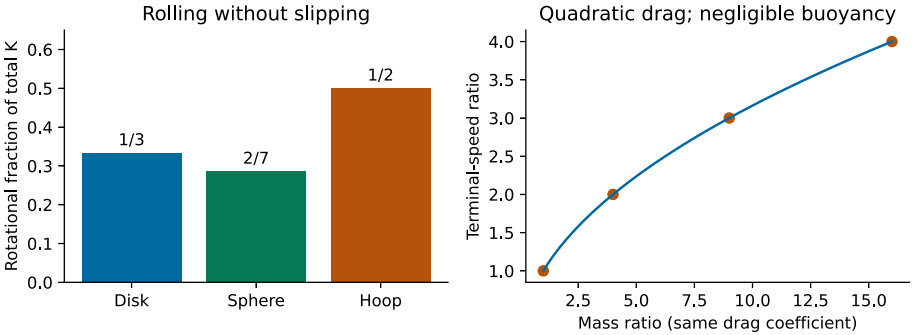
- Uniform rod about its centre: $\frac{1}{12}ML^2$. About one end, by the **parallel axis 平行轴** rule $I = I_{\text{cm}} + Md^2$: $\frac{1}{12}ML^2 + M(L/2)^2 = \frac{1}{3}ML^2$.
- A rod pivoted at one end, at angle θ from the vertical: $\tau = Mg\frac{L}{2}\sin\theta$, so $\alpha = \dfrac{3g\sin\theta}{2L}$.
- Other shapes about their centres: hoop MR^2, disk $\frac{1}{2}MR^2$, solid sphere $\frac{2}{5}MR^2$.

Rolling without slipping

- The final speed after a drop h does not depend on M or R, only on β.
- The share of energy in rotation is $\beta/(1+\beta)$: $1/3$ for a disk, $2/7$ for a sphere, $1/2$ for a hoop.
- In a race down a slope, the smallest β wins: sphere, then disk, then hoop.

$$v^2 = \frac{2gh}{1+\beta}$$

Figure: Rolling without slipping



- Rolling shares energy by $I/(mr^2)$; quadratic drag gives terminal speed $\propto \sqrt{m}$
- Read the axes and units first.

Buoyancy

- A floating object displaces its own weight of fluid. Wood of density 600 kg/m^3 floats in water with 60% of its volume below the surface.
- Count every submerged part. For a composite object, the total displaced volume is set by the total mass, wherever the parts are.

Terminal speed

- Quadratic drag (fast or large objects), $Cv^2 = Mg$: $v_t = \sqrt{Mg/C}$. Doubling the mass multiplies v_t by $\sqrt{2}$.
- Linear drag (slow, small objects in viscous fluids), $bv = Mg$: $v_t \propto M$.
- With buoyancy, replace Mg by $(M - \rho_f V)g$.

Worked example: a falling rod

- **Torque**: $\tau = Mg(L/2)$; **moment of inertia**: $\frac{1}{3}ML^2$.
- $\alpha = \dfrac{3g}{2L} = \dfrac{3(9.8)}{2(1.2)} = 12.3$ rad/s^2.
- The free end starts with acceleration $\alpha L = 14.7$ m/s^2, more than g. A coin resting on that end would be left behind.

Worked example: flipping a composite block

- Weight is unchanged, so the total displaced volume is still $0.60V$.
- The submerged stone now displaces $0.10V$ itself.
- So only $0.50V$ of wood is under water: the block floats higher.

Common errors

- Taking I about the centre when the body turns about a pivot.
- Thinking a heavier or larger ball rolls down faster; only the shape factor β matters.
- Ignoring the submerged parts of a composite object.

Common errors (2)

- Using $v_t \propto M$ for quadratic drag.
- Forgetting buoyancy for objects falling through dense liquids.

Key terms

- **drag**: The resistive force a fluid exerts on a moving object.
- **torque**: The turning effect of a force, $rF\sin\phi$.
- **moment of inertia**: Resistance to angular acceleration, $\sum mr^2$ about an axis.
- **parallel axis**: The rule $I = I_{\text{cm}} + Md^2$ for an axis parallel to one through the centre of mass.

Check yourself

- A hoop and a solid sphere race from rest down the same slope. Which wins, and what is the sphere's v^2 after a drop h?
- With quadratic drag, how does the terminal speed change if the mass is doubled?
- A plastic of density 900 kg/m^3 floats in water. What fraction is submerged?