Collisions, work and oscillator energy

GRE undergraduate preparation · local supported objectives

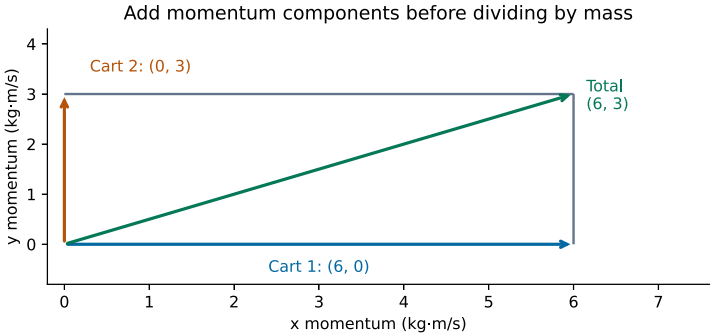
Collisions, work and oscillator energy

- Conserve momentum as a vector, component by component.
- Tell an **elastic collision 弹性碰撞** from an **inelastic collision 非弹性碰撞**.
- Use the **work–energy theorem 动能定理** and spring energy.
- Find oscillator energy, maximum speed and pendulum periods.

Momentum by components

- A 2 kg cart at 3 m/s east and a 1 kg cart at 3 m/s north have total momentum $(6, 3)$ kg m/s.
- If they stick, the 3 kg wreck moves at $(2, 1)$ m/s, about 2.2 m/s.
- Adding the magnitudes, $6 + 3 = 9$ kg m/s, would be wrong.

Figure: Momentum by components



- Add momentum as vectors first, then divide by the total mass
- Read the axes and units first.

Elastic and inelastic collisions

- One-dimensional **elastic collision** with m_2 at rest: $v_1' = \dfrac{m_1 - m_2}{m_1 + m_2} v_1$ and $v_2' = \dfrac{2m_1}{m_1 + m_2} v_1$.
- Equal masses swap velocities; a light ball bounces back off a heavy one.
- In an elastic collision, the relative speed of approach equals the relative speed of separation.

Work–energy theorem and springs

- Doubling the speed quadruples K, so with the same braking force the stopping distance is four times as long.
- A spring stores $U = \tfrac{1}{2}kx^2$.
- Same force F on two springs: $x = F/k$, so $U = F^2/2k$ and the softer spring stores more. Same extension: the stiffer spring stores more.

Oscillator energy

- So $v_{\max} = \omega A$, with $\omega = \sqrt{k/m} = 2\pi f$.
- At $x = A/2$, the potential energy is $E/4$ and the kinetic energy $3E/4$.

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2$$

Pendulums and small angles

- The period does not depend on the mass or (for small swings) the **amplitude** 振幅.
- Four times the length doubles the period. Large amplitudes make the period slightly longer.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Worked example: a two-dimensional sticking collision

- Momentum: $(6,0)+(0,3)=(6,3)$ kg m/s, so $\mathbf{v}=(6,3)/3=(2,1)$ m/s.
- Kinetic energy before: $\tfrac{1}{2}(2)(9)+\tfrac{1}{2}(1)(9)=13.5$ J. After: $\tfrac{1}{2}(3)(2^2+1^2)=7.5$ J.
- So 6.0 J becomes **internal energy**: heat, sound and bent metal.

Worked example: a one-dimensional elastic collision

- $v_1' = \dfrac{1-3}{4}(4.0) = -2.0$ m/s (it bounces back); $v_2' = \dfrac{2}{4}(4.0) = 2.0$ m/s.
- Momentum check: $4.0 = -2.0 + 3.0 \times 2.0$.
- Energy check: $\frac{1}{2}(1)(16) = 8.0$ J before; $\frac{1}{2}(1)(4) + \frac{1}{2}(3)(4) = 8.0$ J after.

Common errors

- Adding momentum magnitudes instead of vectors.
- Assuming kinetic energy is conserved because momentum is.
- Saying the stiffer spring always stores more energy; it depends on whether force or extension is fixed.

Common errors (2)

- Including the mass in the pendulum period.
- Using $T = 2\pi\sqrt{L/g}$ for large swings.

Key terms

- **elastic collision**: A collision in which kinetic energy is conserved.
- **inelastic collision**: A collision in which some kinetic energy becomes **internal energy**.
- **work–energy theorem**: The net work done on a body equals its change in kinetic energy.
- **impulse**: Force multiplied by the time it acts; the change in momentum.

Check yourself

- A 0.50 kg oscillator passes equilibrium at 0.20 m/s. What is its total energy?
- A pendulum is made four times as long. What happens to its period?
- A 10 N force stretches springs of $k = 100$ N/m and $k = 400$ N/m. How much energy does each store?