

Collisions, work and oscillator energy

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

Two carts approach from different directions. After they stick, their momentum can be conserved while their kinetic energy falls. Compute vector momentum and energy loss in sticking collisions. Compare fixed-force springs and use work–energy conditions. Derive small-oscillation periods and oscillator energies.

Choose the system and model

Reasoning

Choose an isolated system during the short collision. With negligible external impulse, conserve total momentum separately in every Cartesian direction. If masses m_1 and m_2 stick, their common velocity is $(m_1 v_1 + m_2 v_2)/(m_1 + m_2)$. Find the magnitude only after adding vectors. Initial kinetic energy is the sum of each $\frac{1}{2}m|v|^2$; final kinetic energy uses the total mass and the common speed. Their difference becomes internal energy or deformation, not missing momentum. A perfectly inelastic collision means sticking, not final rest.

Use the governing relation

Reasoning

The work–energy theorem $\Delta K = \int \mathbf{F} \cdot d\mathbf{r}$ uses net force along displacement. For a constant parallel force over distance s , $\Delta K = Fs$. Speed doubling quadruples kinetic energy. Conservation of mechanical energy is a separate statement requiring no unaccounted nonconservative work. At a spring displacement x , $U = \frac{1}{2}kx^2$. Under the same applied force F , equilibrium extension is F/k and stored energy is $F^2/(2k)$: a stiffer spring stretches less and stores less energy. Under the same extension, it stores more. Identify which condition is held fixed.

Apply the conditions

Reasoning

Near a stable equilibrium x_0 , expand $U \approx U(x_0) + \frac{1}{2}U''(x_0)(x - x_0)^2$; the effective stiffness is positive $U''(x_0)$. The motion obeys $m \ddot{x} = -k_{\text{eff}}(x - x_0)$, with $\omega = \sqrt{k_{\text{eff}}/m}$ and period $2\pi/\omega$. A simple pendulum obeys $\ddot{\theta} + (g/L)\sin\theta = 0$. For small angles $\sin\theta \approx \theta$, so $T = 2\pi\sqrt{L/g}$. This approximation explains period-length scaling; a large amplitude requires a correction, and the mass cancels for an ideal pendulum.

Check the conclusion

Reasoning

For an undamped harmonic oscillator, total energy is $\frac{1}{2}m v^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. At equilibrium all its energy is kinetic; at a turning point its speed is zero and all its energy is potential. Angular frequency is not ordinary frequency: $\omega = 2\pi f$. If the same oscillator has equilibrium speed $v_{\max}$, $A = v_{\max}/\omega$. Distinguish the instant at which speed is measured from the release displacement, and keep SI units when a millijoule answer is requested.

Worked reasoning 1

Reasoning

Two carts stick with negligible external impulse. Add momentum vectors , then divide by total mass. Known: $m_1 = 2 \text{ kg}$, $v_1 = (3,0) \text{ m/s}$; $m_2 = 1 \text{ kg}$, $v_2 = (0,3) \text{ m/s}$.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$\mathbf{v}_f = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{(2 \text{ kg})(3, 0) \text{ m/s} + (1 \text{ kg})(0, 3) \text{ m/s}}{3 \text{ kg}} = (2, 1) \text{ m/s}.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(1\text{ kg})(3\text{ m/s})^2 = 13.5\text{ J}.$$

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

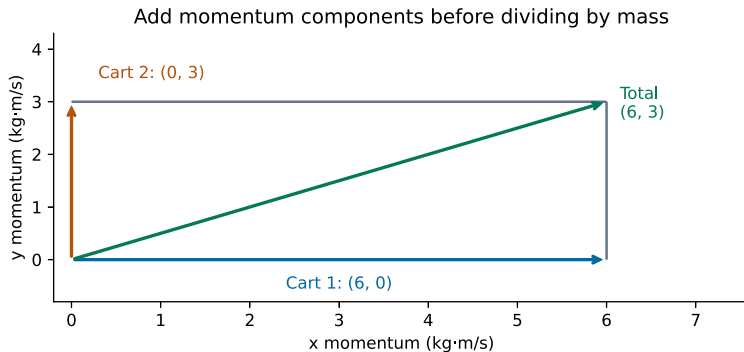
$$K_f = \frac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 = \frac{1}{2}(3 \text{ kg})(5 \text{ m}^2/\text{s}^2) = 7.5 \text{ J}.$$

Worked reasoning 7

Reasoning

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.

Read the original diagram



- Original GRE teaching diagram: Collisions, work and oscillator energy
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

Two equal masses approach with speed u along directions separated by angle θ and stick. Derive final speed and the fraction of initial kinetic energy lost. Check parallel, perpendicular and opposite incoming directions. **[5]**

Independent transfer · reasoning

Reasoning

The sum of equal velocity vectors has magnitude $2u|\cos(\theta/2)|$. Dividing momentum by total mass gives $v_f = u|\cos(\theta/2)|$. $K_i = mu^2$ and $K_f = mv_f^2$, so lost fraction is $\sin^2(\theta/2)$. The three limits are 0, 1/2 and 1. Award vector sum, speed, energy relation, lost fraction and limits. The formula uses $0 \leq \theta \leq \pi$ between directions.

Check assumptions and limits

Reasoning

Do not add incoming speed magnitudes as momenta, or conserve kinetic energy in a sticking collision. Fixed-force and fixed-extension spring comparisons have opposite answers.

Keep the technical distinctions

Reasoning

external impulse
system. turning point
speed is zero.

: Time integral of the net force from outside the chosen
: An extreme oscillator position where its instantaneous

Check yourself

- Define external impulse and give its condition of use.
- Define turning point and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.