Solid-state, nuclear and particle models

GRE undergraduate preparation · local supported objectives

Solid-state, nuclear and particle physics

- Explain conduction with bands, the **band gap** 带隙 and **doping** 掺杂.
- Use Fermi–Dirac occupancy near the **Fermi level** 费米能级.
- Work with decay laws, **activity** 活度 and **half-life** 半衰期.
- Find **binding energy** 结合能 and check conservation laws in reactions.

Energy bands

- Metals have a partly filled band, so they conduct at any temperature.
- Insulators have a full band and a large gap (diamond: about 5.5 eV).
- Semiconductors have a small gap (silicon: 1.1 eV), so heat lifts a few electrons across it; their conductivity rises with temperature.

Doping

- Donors (phosphorus or arsenic in silicon) have one extra electron, which easily enters the conduction band: n-type.
- Acceptors (boron or aluminium) take an electron and leave a hole in the valence band: p-type.
- One phosphorus atom per million silicon atoms gives about 5×10^{22} m^{-3} electrons, far more than pure silicon has at room temperature.

Fermi–Dirac occupancy

- At $T = 0$, f is a step: 1 below μ and 0 above.
- At $E = \mu$, $f = \frac{1}{2}$ at every temperature.
- Heating changes f only in a band about kT wide around μ, which is why most electrons in a metal cannot take up heat.

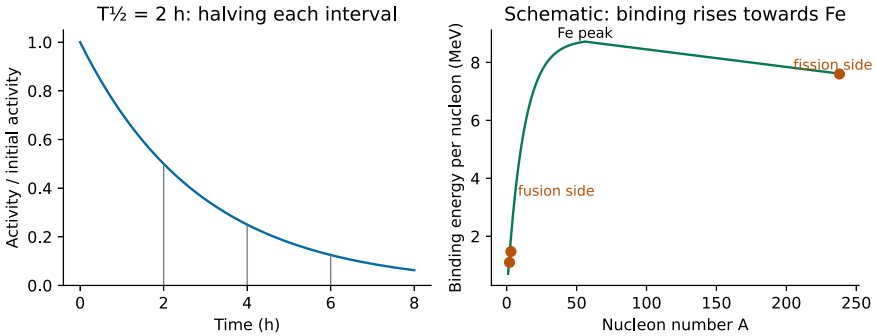
$$f(E) = \frac{1}{e^{(E-\mu)/kT} + 1}$$

Radioactive decay

- After k half-lives, $N/N_0 = 2^{-k}$, and the activity falls by the same factor.
- Activity counts decays per second (becquerel, Bq); N counts nuclei.

$$N = N_0\, e^{-\lambda t}, \qquad A = \lambda N, \qquad t_{1/2} = \frac{\ln 2}{\lambda}$$

Figure: Radioactive decay



- Each **half-life** halves the activity; energy is released by moving towards the iron peak, by fusion from the left or fission from the right
- Read the axes and units first.

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Binding energy

- Stability follows the binding energy per nucleon, which peaks near iron at about 8.8 MeV.
- Fusing light nuclei or splitting heavy ones moves towards the peak and releases energy.

Conservation laws in reactions

- Beta-minus decay is $n \to p + e^{-} + \overline{\nu}_e$. The antineutrino keeps lepton number at zero and shares the energy, which is why the electron energies form a continuous spectrum.
- Protons and neutrons have baryon number $+1$; electrons and neutrinos have electron-lepton number $+1$; antiparticles have the opposite sign.

Worked example: a decaying sample

- After 6.0 h (three half-lives): $A = 800/2^3 = 100$ Bq.
- Decay constant: $\lambda = \ln 2/(7200\ \text{s}) = 9.6 \times 10^{-5}\ \text{s}^{-1}$.
- Nuclei at the start: $N = A/\lambda = 800/(9.6 \times 10^{-5}) = 8.3 \times 10^{6}$.

Worked example: binding energy of helium-4

- Separate particles: $2(1.007276) + 2(1.008665) = 4.031882$ u.
- Mass defect: 0.030376 u, so $E_B = 0.030376 \times 931.5 = 28.3$ MeV.
- Per nucleon: $28.3/4 = 7.07$ MeV, which is why helium-4 is so stable for a light nucleus.

Common errors

- Thinking more electrons always means better conduction; a full band carries no current.
- Saying $f(\mu) = 1$ at absolute zero; $f(\mu) = \frac{1}{2}$.
- Confusing activity with the number of nuclei.

Common errors (2)

- Judging stability by total binding energy instead of binding energy per nucleon.
- Writing beta decay without the antineutrino.

Key terms

- **band gap**: The range of energies with no electron states between the valence and conduction bands.
- **doping**: Adding small amounts of impurity to change the number of charge carriers.
- **Fermi level**: The energy at which a state has a 50% chance of being occupied.
- **activity**: The number of decays per second in a sample.

Check yourself

- What fraction of a sample remains after five half-lives?
- What is the Fermi–Dirac occupancy of a state at $E = \mu + kT$?
- A π^- decays to a μ^- and one more particle. Which?