Relativity, laboratory methods and specialised topics

GRE undergraduate preparation · local supported objectives

Relativity and laboratory methods

- Use the **Lorentz factor 洛伦兹因子** for time, length, energy and momentum.
- Remember that kinetic energy is not $\frac{1}{2}mv^2$ at high speed.
- Combine uncertainties and tell **precision 精密度** from **accuracy 准确度**.
- Read straight lines from log plots.

Time dilation and length contraction

- A moving clock runs slow: $\Delta t = \gamma\,\Delta\tau$, where $\Delta\tau$ is the proper time in the clock's own frame.
- A moving rod is shorter along its motion: $L = L_0/\gamma$.
- Muons made high in the atmosphere live 2.2 μs at rest, yet many reach the ground because their lifetime is dilated in our frame.

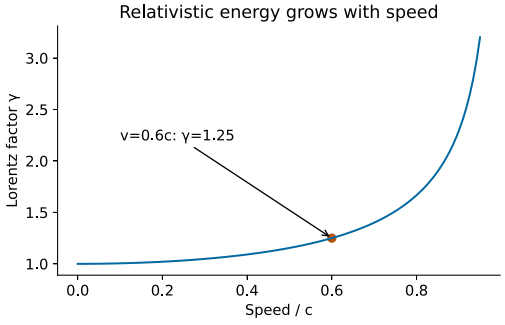
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Relativistic energy and momentum

- Kinetic energy is $K = (\gamma - 1)mc^2$. At $v = 0.6c$, $\frac{1}{2}mv^2 = 0.18\,mc^2$, but $K = 0.25\,mc^2$.
- For a photon, $m = 0$ and $E = pc$.
- Mass is not "relativistic mass"; m is the same in every frame.

$$E = \gamma mc^2, \qquad p = \gamma mv, \qquad E^2 = (pc)^2 + (mc^2)^2$$

Figure: Relativistic energy and momentum



- Identify the axes, units, directions and the stated model.

Combining uncertainties

- For a sum or difference, add the absolute uncertainties in **quadrature**.
- For a product or quotient, add the relative (percentage) uncertainties in quadrature: 3% and 4% give 5%.
- For $y = x^n$, multiply the relative uncertainty by $|n|$. Simple addition gives a worst case, not the standard uncertainty.

Precision, accuracy and counting

- Averaging N readings shrinks the random error by $1/\sqrt{N}$ but leaves a bias unchanged.
- When counting random events (radioactive decays, photons), N counts have an uncertainty of $\sqrt{N}$, a relative uncertainty of $1/\sqrt{N}$.

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Log plots

- If $y = Ax^n$, then $\log y = \log A + n \log x$: on log–log axes the line has slope n.
- If $y = Ae^{kx}$, then $\ln y = \ln A + kx$: a semi-log plot is straight with slope k.
- Read the slope from the logs of the values, not from the printed tick labels.

Worked example: an electron at 0.6c

- $\gamma = 1/\sqrt{1-0.36} = 1.25$, so $E = 0.639$ MeV and $K = 0.25 \times 0.511 = 0.128$ MeV.
- Momentum: $pc = \gamma mvc = 1.25 \times 0.6 \times 0.511 = 0.383$ MeV.
- Check: $E^2 = 0.408$ and $(pc)^2 + (mc^2)^2 = 0.147 + 0.261 = 0.408$ MeV^2.

Worked example: uncertainty in g

- The T^2 term doubles its relative uncertainty: $2 \times 0.50\% = 1.0\%$.
- In **quadrature**: $\sqrt{1.0^2 + 1.0^2} = 1.4\%$.
- A counter records 400 decays: 400 ± 20, or 5%. For 1%, it needs $1/\sqrt{N} = 0.01$, so 10,000 counts.

Common errors

- Using $\frac{1}{2}mv^2$ for kinetic energy at speeds near c.
- Applying **time dilation** the wrong way round: the proper time is the shortest.
- Adding uncertainties linearly when they are independent.

Common errors (2)

- Hoping that more readings will remove a **systematic error**.
- Reading a log–log slope from the tick labels instead of their logarithms.

Key terms

- **time dilation**: A moving clock runs slow by the factor γ compared with clocks at rest.
- **Lorentz factor**: $\gamma = 1/\sqrt{1 - v^2/c^2}$.
- **rest energy**: The energy mc^2 a particle has when it is at rest.
- **precision**: How closely repeated measurements agree.

Check yourself

- What is γ at $v = 0.8c$?
- Two independent relative uncertainties are 6% and 8%. What is the uncertainty of their product?
- A log–log plot of y against x is a straight line of slope -2. How does y depend on x?