

Quantum mechanics and atomic physics

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A particle in a box cannot have zero ground-state kinetic energy. Its confinement changes what states are allowed. Use wavefunctions, operators and measurement probabilities Solve standard bound-state models and tunnelling reasoning Apply angular momentum, spin and atomic spectral principles

Choose the system and model

Reasoning

A state wavefunction must satisfy the model's boundary conditions and have total probability one: integrate $|\psi(x)|^2$ over the allowed domain. For orthonormal states ϕ_j , a superposition $\psi = \sum c_j \phi_j$ has norm squared $\sum |c_j|^2$ because the cross terms integrate to zero. Thus an equal superposition of N orthonormal states has coefficient magnitude $1/\sqrt{N}$. The probability of finding one constituent energy is $|c_j|^2$, not c_j . Relative phases may affect position probabilities even when energy probabilities are unchanged.

Use the governing relation

Reasoning

An observable is represented by a Hermitian operator. Its eigenvalues are real, and a normalised state gives expectation value $\langle A \rangle = \int \psi^* A \psi \, dx$. The expectation is an average over repeated preparations, not necessarily the result of one measurement. If A satisfies a polynomial identity, apply it to an eigenstate: for $A^4 = I$, a real eigenvalue must be $+1$ or -1 . A complex fourth root is excluded by Hermiticity even though it solves the polynomial.

Apply the conditions

Reasoning

For an infinite well $0 < x < L$, $\psi(0) = \psi(L) = 0$ allows sine modes $\sin(n\pi x/L)$, with $n=1, 2, \dots$. Their energies are $E_n = n^2 \pi^2 \hbar^2 / (2mL^2)$, so the ground state is not zero and doubling L divides every energy by four. The equivalent h expression is $n^2 h^2 / (8mL^2)$; never substitute h for $\hbar$ without its 2π factor. A finite barrier instead permits exponential tails, and a travelling wave in a classically allowed region has oscillatory rather than exponentially decaying spatial dependence.

Check the conclusion

Reasoning

Operators commute when $[A,B]=AB-BA=0$; compatible observables can have a common eigenbasis. Since kinetic energy $p^2/(2m)$ is a function of p , it commutes with p . A position-dependent potential generally makes $H=p^2/(2m)+V(x)$ fail to commute with p . Angular momentum obeys $[J_x,J_y]=i\hbar J_z$ and cyclic permutations, while J^2 commutes with each component. Atomic transitions exchange positive photon energy equal to a level difference; angular momentum and mechanism-specific selection rules restrict which transitions occur.

Worked reasoning 1

Reasoning

An infinite well extends from 0 to L . Vanishing wavefunction at both ends gives

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$\psi_n = \sqrt{2/L} \sin(n\pi x/L), \quad E_n = n^2 \pi^2 \hbar^2 / (2mL^2).$$

Worked reasoning 3

Reasoning

Prepare $\psi = (\psi_1 + i\psi_2)/\sqrt{2}$. Orthogonality makes its norm one. Energy measurements give E_1 or $4E_1$ with equal probability:

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

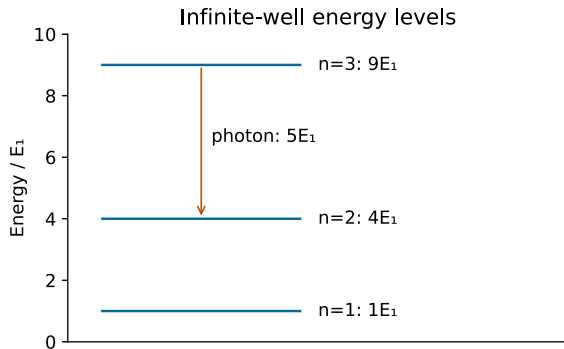
$$\langle E \rangle = \frac{1}{2}E_1 + \frac{1}{2}(4E_1) = \frac{5}{2}E_1.$$

Worked reasoning 5

Reasoning

This average is not an allowed single energy result. The relative phase changes in time, so a superposition of different energies is not a stationary state.

Read the original diagram



- Original GRE teaching diagram: Quantum mechanics and atomic physics
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

In the same well a normalised state is $(\psi_1 + 2\psi_3)/\sqrt{5}$. Find its possible measured energies and probabilities, mean energy and energy variance. Is the mean an allowed result of one measurement? **[5]**

Independent transfer · reasoning

Reasoning

Results are E_1 with probability $1/5$ and $9E_1$ with probability $4/5$. $\langle E \rangle = 37E_1/5$; $\langle E^2 \rangle = 65E_1^2$. $\text{Var}(E) = 65E_1^2 - (37E_1/5)^2 = 256E_1^2/25$. The mean $7.4E_1$ is not a well eigenvalue. Award results, probabilities, mean, second-moment/variance calculation and interpretation.

Check assumptions and limits

Reasoning

A stationary state can have a nonzero energy even though its probability density is time-independent.

Keep the technical distinctions

Reasoning

normalisation : Making total probability equal to one. operator : A mathematical action representing an observable.

Check yourself

- Define normalisation and give its condition of use.
- Define operator and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.