Electromagnetism

GRE undergraduate preparation · local supported objectives

Electromagnetism

- Get the field from the potential, and know that a value of V alone says nothing about $\mathbf{E}$.
- Use **Gauss's law** 高斯定律 only where symmetry makes it work.
- Solve circuits with Kirchhoff's rules and RC decay.
- Apply Faraday's law, **Lenz's law** and the properties of electromagnetic waves.

Field and potential

- Midway between $+q$ and $-q$, $V = 0$, but the field is strong and points from $+q$ to $-q$.
- Midway between two equal charges $+q$, $E = 0$, but $V \neq 0$.
- Point charge: $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ and $V = \dfrac{q}{4\pi\varepsilon_0 r}$. A charge q moved through ΔV gains energy $q\,\Delta V$.

Gauss's law needs symmetry

- Sphere (outside): $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$. Long line: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. Large plane: $E = \dfrac{\sigma}{2\varepsilon_0}$.
- Inside a uniformly charged shell, $E = 0$, and V is constant at its surface value, not zero.
- Without symmetry (a cube with a charge off-centre), **Gauss's law** still gives the total **flux**, but not E.

Conductors in equilibrium

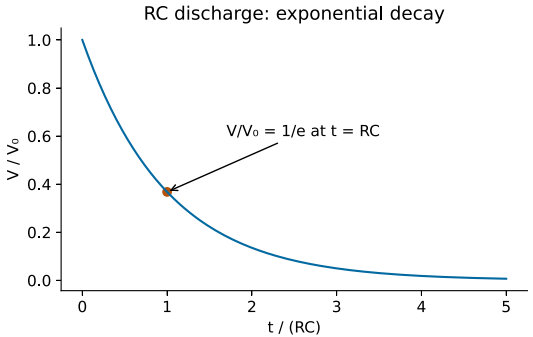
- Any excess charge sits on the outer surface; the whole **conductor** is at one potential.
- Just outside the surface, the field is perpendicular to it with size σ/ε_0.
- A closed metal shell screens its inside from outside charges. This is a Faraday cage.

Circuits and RC decay

- After one **time constant**, $V = V_0/e \approx 0.37\,V_0$; after 5τ, less than 1% is left.
- The time to fall to half is $\tau \ln 2 \approx 0.69\,\tau$.
- Energy stored in a capacitor: $\frac{1}{2}CV^2$. Halving the voltage leaves a quarter of the energy.

$$V(t) = V_0\, e^{-t/RC}$$

Figure: Circuits and RC decay



- After one **time constant** RC the voltage has fallen to $1/e \approx 0.37$ of its start
- Read the axes and units first.

Induction and electromagnetic waves

- A rod of length L moving at v across a field B has $\mathcal{E} = BLv$.
- In vacuum, Maxwell's equations give waves travelling at $c = 1/\sqrt{\mu_0 \varepsilon_0}$, with $E = cB$ and $\mathbf{E} \perp \mathbf{B} \perp$ the direction of travel.
- The energy flow is $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$; the average intensity is $E_0^2/(2\mu_0 c)$.

Worked example: an RC discharge

- **Time constant**: $\tau = RC = (1.0 \times 10^{5})(1.0 \times 10^{-5}) = 1.0$ s.
- After 2.0 s: $V = 12\,e^{-2} = 1.6$ V.
- Time to reach 3.0 V: $3 = 12\,e^{-t/1.0}$, so $t = \ln 4 = 1.39$ s.

Worked example: a line of charge

- Symmetry: the field is radial and has the same size on a coaxial cylinder, so **Gauss's law** applies.
- Gauss: $E(2\pi rL) = \lambda L/\varepsilon_0$, so $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2.0\times10^{-9}}{2\pi(8.85\times10^{-12})(0.10)} = 360$ N/C.
- Check: doubling r halves E for a line, but quarters it for a point charge.

Common errors

- Concluding $E = 0$ where $V = 0$, or $V = 0$ where $E = 0$.
- Using **Gauss's law** to find E for a shape without symmetry.
- Taking the potential inside a charged shell as zero; it equals the surface value.

Common errors (2)

- Getting the direction of an induced current from the field instead of from the change in **flux**.
- Forgetting that energy in a capacitor goes as V^2.

Key terms

- **conductor**: A material with charges that move freely.
- **Gauss's law**: The **flux** of $\mathbf{E}$ through a closed surface equals the enclosed charge divided by ε_0.
- **electric potential**: **Electric potential** energy per unit charge.
- **flux**: The amount of field passing through a surface.

Check yourself

- What is the field 0.50 m from a 1.0 nC point charge?
- A capacitor discharges through $R = 1.0$ MΩ with $C = 2.0$ μF. How long until the voltage has halved?
- A light wave has $E_0 = 300$ V/m. What is B_0?