

Classical mechanics

GRE undergraduate preparation · local supported objectives

Choose the physical question

Reasoning

A planet accelerates even when its speed is constant: velocity includes direction. Apply Newtonian, Lagrangian and Hamiltonian descriptions Analyse rotation, central forces and oscillations Use conservation laws with their conditions

Choose the system and model

Reasoning

Newton's second law is a vector statement. Choose coordinates and identify constraints before components.

Use the governing relation

Reasoning

Energy conservation applies when work from nonconservative forces is accounted for; momentum conservation needs zero net external impulse.

Apply the conditions

Reasoning

For small oscillations, linearise around a stable equilibrium. A restoring term proportional to displacement gives harmonic motion.

Check the conclusion

Reasoning

Lagrange's equations use $L=T-V$ and generalised coordinates. This undergraduate formalism is distinct from school-level force substitution.

Worked reasoning 1

Reasoning

Choose an inertial frame and a coordinate x measured from spring equilibrium. For a conservative spring, the Lagrangian avoids solving constraint forces.

Worked relation 2

Reasoning

Apply this relation only under the conditions just stated.

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2.$$

Worked relation 4

Reasoning

Apply this relation only under the conditions just stated.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \quad m\ddot{x} + kx = 0.$$

Worked reasoning 5

Reasoning

With $m = 2.0 \text{ kg}$ and $k = 18 \text{ N/m}$,

Worked relation 6

Reasoning

Apply this relation only under the conditions just stated.

$$\omega = \sqrt{k/m} = \sqrt{(18 \text{ N/m})/(2.0 \text{ kg})} = 3.0 \text{ rad/s.}$$

Worked relation 8

Reasoning

Apply this relation only under the conditions just stated.

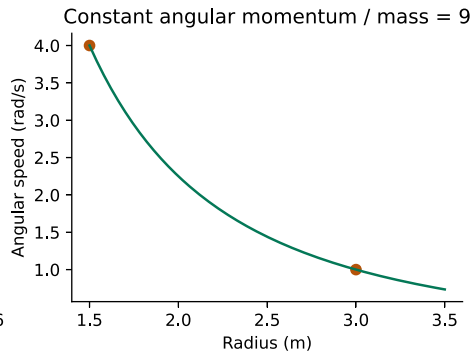
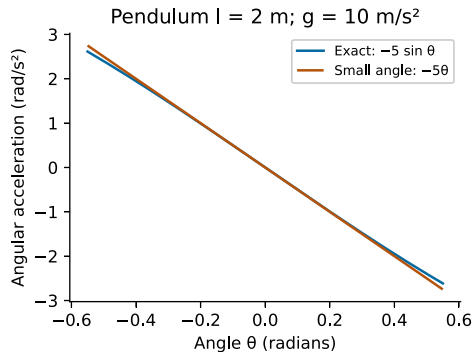
$$T = 2\pi/\omega = 2\pi/(3.0 \text{ rad/s}) = 2.09 \text{ s}.$$

Worked reasoning 9

Reasoning

Positive stiffness gives stable motion. The units of k/m are inverse seconds squared.

Read the original diagram



- Original GRE teaching diagram: Classical mechanics
- Identify the axes, units, directions and the stated model.

Independent transfer · attempt

Reasoning

A bead of mass m moves without friction along a straight rail. Its potential is $V(x) = ax^4 - bx^2$, with a, b positive. Find every equilibrium, classify stability, and derive the small-oscillation frequency at each stable equilibrium. State why the frequency formula cannot describe motion near $x = 0$. ****[6]****

Independent transfer · reasoning

Reasoning

Force is $F = -V' = -4ax^3 + 2bx$. Equilibrium requires $x = 0$ or $x^2 = b/(2a)$.

Curvature is $V'' = 12ax^2 - 2b$: it is $-2b$ at zero and $4b$ at either outer point. Zero is unstable; both outer points are stable. Write $x = x_0 + \xi$ and retain terms linear in ξ : $m\ddot{\xi} = -4b\xi$, so $\omega = \sqrt{4b/m}$. At zero the displacement grows rather than oscillates.

The approximation requires $|\xi|$ small compared with $\sqrt{b/(2a)}$. Award equilibrium, curvature, classification, linearisation, frequency and condition: one mark each.

Check assumptions and limits

Reasoning

Conservation of kinetic energy does not hold in every collision, even when momentum is conserved.

Keep the technical distinctions

Reasoning

Lagrangian : Kinetic energy minus potential energy for the standard mechanical system. angular momentum : The moment of linear momentum about a reference point.

Check yourself

- Define Lagrangian and give its condition of use.
- Define angular momentum and give its condition of use.
- Redo the original transfer with a changed parameter; check a limiting case.