Course Companion

# GRE Subject Test · GRE Physics

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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# Classical mechanics

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A planet accelerates even when its speed is constant: velocity includes direction.

Prerequisites: 47.

- Apply Newtonian, Lagrangian 拉格朗日量 and Hamiltonian descriptions
- Analyse rotation, central forces and oscillations
- Use conservation laws with their conditions

## Choose the system and model

Newton's second law is a vector statement. Choose coordinates and identify constraints before components.

## Use the governing relation

Energy conservation applies when work from nonconservative forces is accounted for; momentum conservation needs zero net external impulse.

## Apply the conditions

For small oscillations, linearise around a stable equilibrium. A restoring term proportional to displacement gives harmonic motion.

## Check the conclusion

Lagrange's equations use L=T−V and generalised coordinates. This undergraduate formalism is distinct from school-level force substitution.

## Worked method

Choose an inertial frame and a coordinate x measured from spring equilibrium. For a conservative spring, the Lagrangian avoids solving constraint forces.

$$L = \tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}kx^2.$$

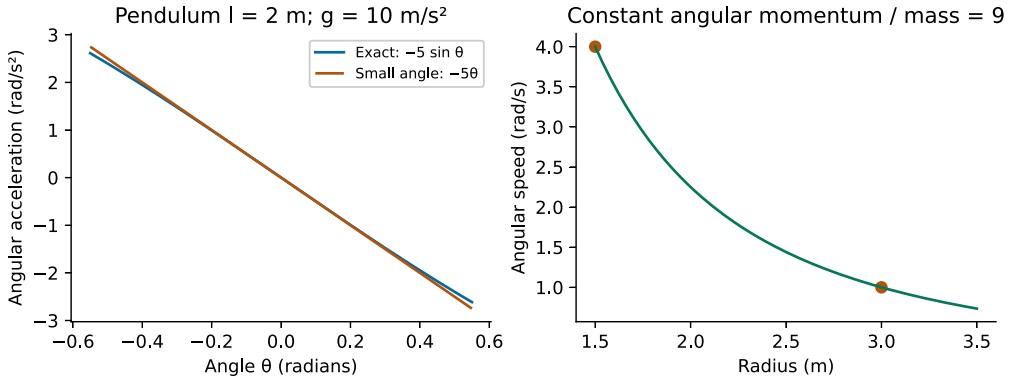
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \qquad m\ddot{x} + kx = 0.$$

With m = 2.0 kg and k = 18 N/m,

$$\omega = \sqrt{k/m} = \sqrt{(18\,\mathrm{N/m})/(2.0\,\mathrm{kg})} = 3.0\,\mathrm{rad/s}.$$

$$T = 2\pi/\omega = 2\pi/(3.0\,\mathrm{rad/s}) = 2.09\,\mathrm{s}.$$

Positive stiffness gives stable motion. The units of k/m are inverse seconds squared.




*Small angles make the pendulum linear ($\theta \approx -5\theta$); constant angular momentum makes $\omega$ fall as $1/r^2$*

## Check conditions and vocabulary

Conservation of kinetic energy does not hold in every collision, even when momentum is conserved.

**Lagrangian**: Kinetic energy minus potential energy for the standard mechanical system.

**angular momentum** 角动量: The moment of linear momentum about a reference point.



# Electromagnetism

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Zero electric potential 电势 at a point does not imply zero electric field there.

Prerequisites: 13, 14, 15, 29, 30.

- Use electrostatic potentials, fields and Gauss's law
- Analyse circuits, magnetism and induction
- Apply Maxwell equations and electromagnetic wave relationships

## Choose the system and model

Electric field is the negative gradient of potential. A value and a spatial change are different quantities.

## Use the governing relation

Use Gauss's law with symmetry. A known total flux 通量 alone does not give the field at every surface point.

## Apply the conditions

Kirchhoff's laws track charge and energy in circuits. Induced emf follows changing magnetic flux with the Lenz-law sign.

## Check the conclusion

Maxwell equations link electric and magnetic fields. In vacuum a wave has E/B=c and carries energy.

## Worked method

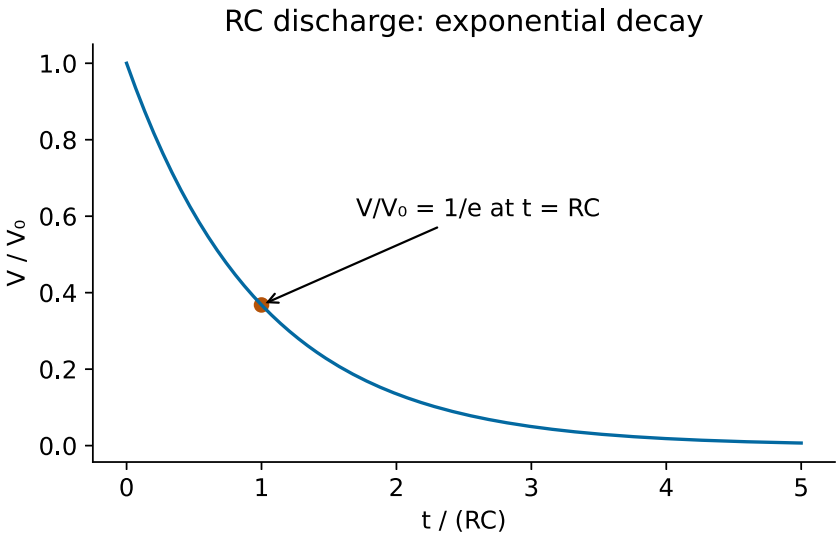
A spherical shell has radius R and uniform charge Q. Choose zero potential at infinity. Spherical symmetry lets Gauss's law 高斯定律 determine the field outside.

$$4\pi r^2 E = Q/\epsilon_0, \qquad E = Q/(4\pi\epsilon_0 r^2) \quad (r > R).$$

Inside, enclosed charge is zero and symmetry gives E = 0. Continuity gives

$$V(r) = \begin{cases} Q/(4\pi\epsilon_0 R) & r \le R, \\ Q/(4\pi\epsilon_0 r) & r > R. \end{cases}$$

The potential is nonzero and constant inside. A zero gradient, rather than a zero value, gives a zero field. The radial field jumps at the charged surface.




*After one time constant RC the voltage has fallen to 1/e ≈ 0.37 of its start*

## Check conditions and vocabulary

Do not use Gauss's law to claim a uniform field on an arbitrary nonsymmetric surface.

**flux**: The surface integral of a field's normal component.

**potential**: Potential energy per unit charge.



# Waves and optics

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Two individually bright waves can cancel at a point if their phases oppose.

Prerequisites: 16, 17, 18, 31, 32, 33.

- Analyse wave superposition and interference
- Use diffraction 衍射, polarisation and optical imaging
- Relate phase, frequency and dispersion

## Choose the system and model

Add wave amplitudes, then calculate intensity. Intensity is proportional to squared amplitude for the same medium.

## Use the governing relation

A path difference gives phase difference 相位差 $2\pi\Delta/\lambda$. Coherent sources need stable relative phase.

## Apply the conditions

For a single slit, minima satisfy a sin $\theta$=m$\lambda$ for nonzero integers m. A double slit has a different interference condition.

## Check the conclusion

Geometric optics needs a stated sign convention. Wave effects matter when apertures are comparable to wavelength.

## Worked method

Coherent waves have a stable phase difference. Add amplitudes before squaring.

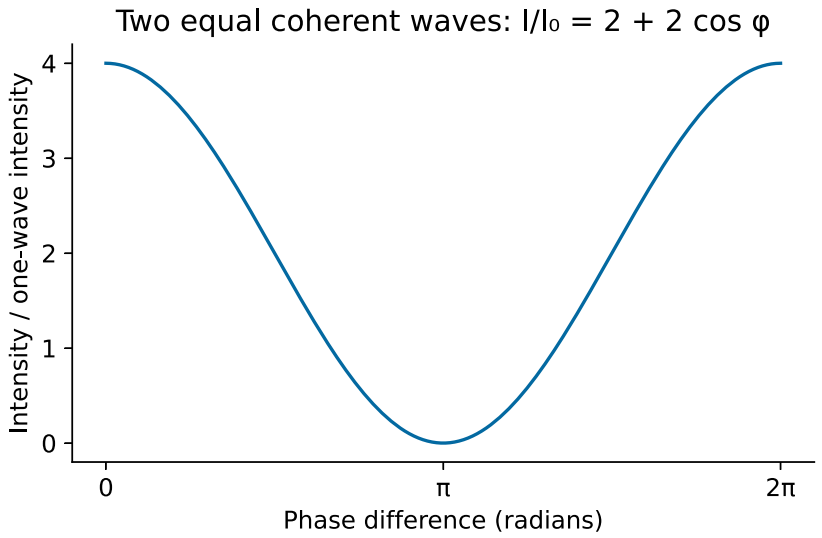
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta.$$

For equal individual intensities I0 and a path difference of one quarter wavelength,

$$\delta = 2\pi\Delta/\lambda = 2\pi(\lambda/4)/\lambda = \pi/2.$$

$$I = 2I_0(1+\cos\delta) = 2I_0(1+\cos(\pi/2)) = 2I_0.$$

This is neither a maximum nor a minimum. Random relative phase removes the cross term in a long time average; stable coherence 相干性 is a physical condition, not a drawing convention.




*$I = 2I_0(1 + \cos\varphi)$: four times one wave in phase, zero in antiphase*

## Check conditions and vocabulary

The central single-slit maximum is not obtained by setting m=0 in the minima condition.

**coherence**: A stable phase relationship between waves.

**diffraction**: Wave spreading and interference caused by an aperture or obstacle.



# Thermodynamics and statistical mechanics

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Two engines can obey energy conservation, yet only one obeys the entropy 熵 limit on efficiency.

Prerequisites: 19, 20, 34, 35, 36.

- Apply the first and second laws with a sign convention
- Relate ensembles, Boltzmann factors and entropy
- Analyse ideal gases, heat engines and quantum statistics

## Choose the system and model

State the first-law sign convention: ΔU=Q−W when W is work done by the system.

## Use the governing relation

For a reversible engine between reservoirs, efficiency is bounded by 1−Tc/Th using absolute temperatures.

## Apply the conditions

Boltzmann weights are exp(−E/kT). Probabilities require division by the partition function 配分函数.

## Check the conclusion

Distinguish Maxwell–Boltzmann, Bose–Einstein and Fermi–Dirac assumptions. Fermions cannot share an identical single-particle state.

## Worked method

Use work W done by the gas as positive. The first law 热力学第一定律 is

$$\Delta U = Q - W.$$

For reversible isothermal expansion of an ideal gas, U depends only on T, so

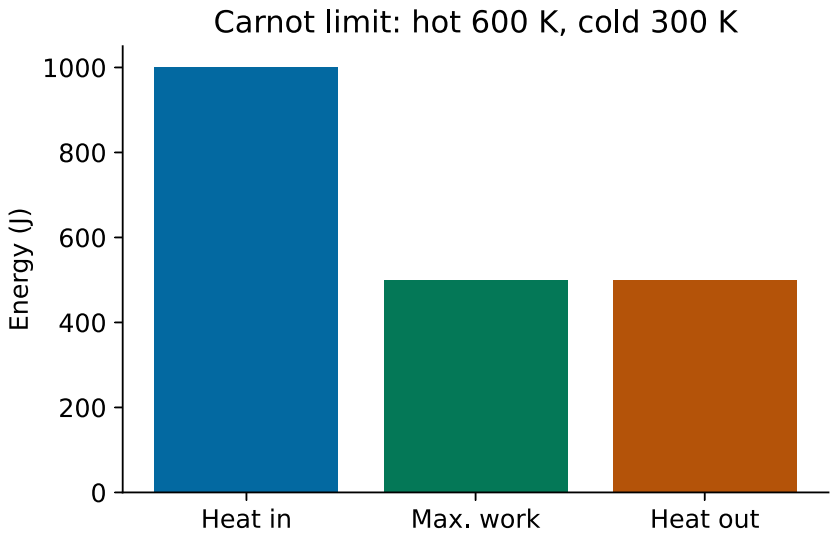
$$W = \int_{V_i}^{V_f} p\,dV = nRT\ln(V_f/V_i), \qquad Q = W.$$

For n = 0.50 mol, T = 300 K and doubled volume,

$$W = nRT\ln 2 = (0.50\,\text{mol})(8.314\,\text{J/(mol K)})(300\,\text{K})\ln 2 = 864\,\text{J}.$$

$$\Delta S = Q_{rev}/T = (864\,\text{J})/(300\,\text{K}) = 2.88\,\text{J/K}.$$

The gas expands and receives heat. The reservoir loses the same entropy in this reversible limit.




*Carnot efficiency 1 − 300/600 = 1/2 caps the work at half the heat input*

## Check conditions and vocabulary

Celsius temperature ratios cannot be used in the Carnot efficiency formula.

**entropy**: A state quantity connected to multiplicity and reversible heat transfer.

**partition function**: The sum of statistical weights used to normalise probabilities.



# Quantum mechanics and atomic physics

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A particle in a box cannot have zero ground-state kinetic energy. Its confinement changes what states are allowed.

Prerequisites: 24, 37, 38, 39.

- Use wavefunctions, operators and measurement probabilities
- Solve standard bound-state models and tunnelling reasoning
- Apply angular momentum, spin and atomic spectral principles

## Choose the system and model

A state wavefunction 波函数 must satisfy the model's boundary conditions and have total probability one: integrate $|\psi(x)|^2$ over the allowed domain. For orthonormal states $\varphi$_j, a superposition $\psi=\Sigma c_j\varphi_j$ has norm squared $\Sigma|c_j|^2$ because the cross terms integrate to zero. Thus an equal superposition of N orthonormal states has coefficient magnitude 1/sqrt(N). The probability of finding one constituent energy is $|c_j|^2$, not c_j. Relative phases may affect position probabilities even when energy probabilities are unchanged.

## Use the governing relation

An observable is represented by a Hermitian operator 算符. Its eigenvalues are real, and a normalised state gives expectation value $\langle A\rangle=\int\psi^*\,A\psi\,dx$. The expectation is an average over repeated preparations, not necessarily the result of one measurement. If A satisfies a polynomial identity, apply it to an eigenstate: for $A^4=I$, a real eigenvalue must be $+1$ or $-1$. A complex fourth root is excluded by Hermiticity even though it solves the polynomial.

## Apply the conditions

For an infinite well $0<x<L$, $\psi(0)=\psi(L)=0$ allows sine modes $\sin(n\pi x/L)$, with $n=1,2,\ldots$. Their energies are $E_n=n^2\pi^2\hbar^2/(2mL^2)$, so the ground state is not zero and doubling L divides every energy by four. The equivalent h expression is $n^2h^2/(8mL^2)$; never substitute h for $\hbar$ without its $2\pi$ factor. A finite barrier instead permits exponential tails, and a travelling wave in a classically allowed region has oscillatory rather than exponentially decaying spatial dependence.

## Check the conclusion

Operators commute when $[A,B]=AB-BA=0$; compatible observables can have a common eigenbasis. Since kinetic energy $p^2/(2m)$ is a function of p, it commutes with p. A position-dependent potential generally makes $H=p^2/(2m)+V(x)$ fail to commute

with p. Angular momentum obeys [J_x,J_y]=iℏJ_z and cyclic permutations, while $J^2$ commutes with each component. Atomic transitions exchange positive photon energy equal to a level difference; angular momentum and mechanism-specific selection rules restrict which transitions occur.

## Worked method

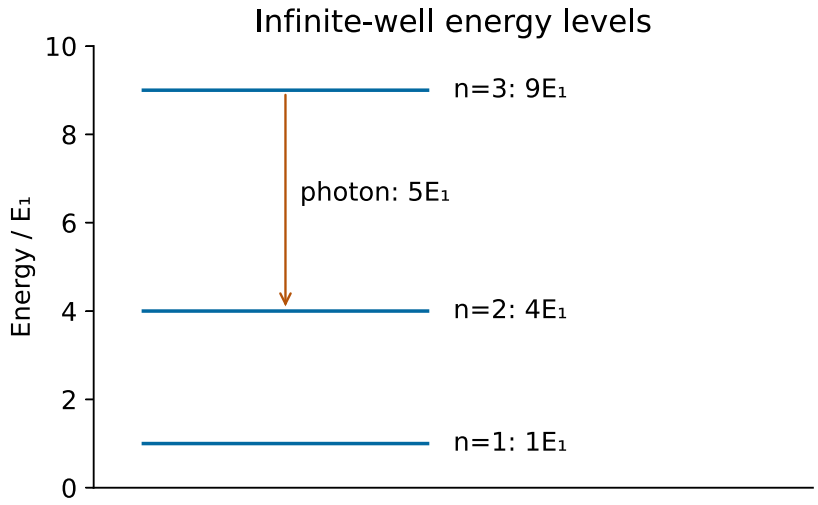
An infinite well extends from 0 to L. Vanishing wavefunction at both ends gives

$$\psi_n = \sqrt{2/L}\sin(n\pi x/L), \qquad E_n = n^2\pi^2\hbar^2/(2mL^2).$$

Prepare $\psi = (\psi_1 + i\psi_2)/\sqrt{2}$. Orthogonality makes its norm one. Energy measurements give E1 or 4E1 with equal probability:

$$\langle E\rangle = \tfrac{1}{2}E_1 + \tfrac{1}{2}(4E_1) = \tfrac{5}{2}E_1.$$

This average is not an allowed single energy result. The relative phase changes in time, so a superposition of different energies is not a stationary state.




*Levels scale as $n^2$, so the 3 → 2 photon carries $9E_1 - 4E_1 = 5E_1$*

## Check conditions and vocabulary

A stationary state can have a nonzero energy even though its probability density is time-independent.

**normalisation** 归一化: Making total probability equal to one.

**operator**: A mathematical action representing an observable.



# Relativity, laboratory methods and specialised topics

GRE Subject Test · GRE Physics

A precise instrument can give consistently wrong results when its calibration is shifted.

Prerequisites: 21, 22, 42, 43, 44.

- Apply special-relativistic energy and spacetime relations
- Propagate measurement uncertainty and analyse experimental data
- Recognise nuclear, particle, condensed-matter and astrophysical concepts

## Choose the system and model

Special relativity uses γ=1/sqrt(1−v²/c²). Rest energy is $mc^2$ and total energy is $\gamma mc^2$.

## Use the governing relation

For independent small uncertainties, combine contributions in quadrature; correlated uncertainties require covariance 协方差 terms.

## Apply the conditions

Random scatter affects precision 精密度; calibration bias affects accuracy. A log plot can reveal a power law.

## Check the conclusion

Nuclear, particle, condensed-matter and astrophysics questions require undergraduate vocabulary and models. School physics alone is not a complete preparation course.

## Worked method

Proper time 固有时 is measured between events at one position in a frame. For a particle speed 0.60c,

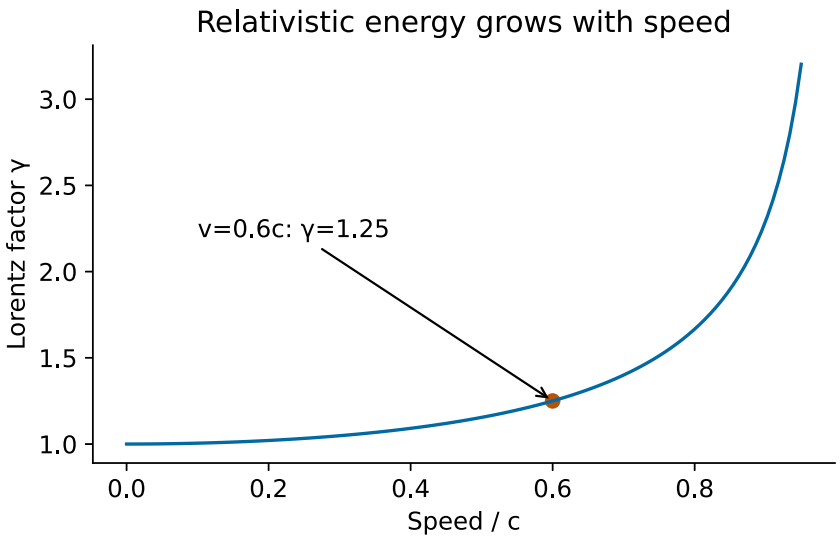
$$\gamma = (1 - v^2/c^2)^{-1/2} = (1 - 0.60^2)^{-1/2} = 1.25.$$

If its mean proper lifetime is 4.0 microseconds,

$$\tau = \gamma\tau_0 = 1.25(4.0\,\mu\text{s}) = 5.0\,\mu\text{s}.$$

$$\bar{d} = v\tau = (0.60)(3.00\times10^8\,\text{m/s})(5.0\times10^{-6}\,\text{s}) = 900\,\text{m}.$$

A lifetime is a mean of a distribution; it does not predict an individual decay time.




*At $v = 0.6c$, $\gamma = 1/\sqrt{(1 - 0.36)} = 1.25$*

## Check conditions and vocabulary

Adding percentage uncertainties linearly gives a worst-case bound, not the independent statistical standard uncertainty.

**precision**: Closeness of repeated measurements to each other.

**covariance**: A measure of joint variation used for correlated uncertainties.



# Atomic spectra and selection rules

GRE Subject Test · GRE Physics

Not every energy difference produces a strong electric-dipole 电偶极 line.

Prerequisites: 23, 40, 41.

- Relate atomic energies to spectral lines
- Apply angular-momentum quantum numbers
- Distinguish allowed and forbidden electric-dipole transitions

## Choose the system and model

For hydrogen-like one-electron ions, the bound energies scale as $-13.6\ Z^2/n^2$ eV in the simplest nonrelativistic model. Emission requires a downward transition and positive photon energy.

## Use the governing relation

Orbital angular momentum has l=0,…,n−1 and m_l=−l,…,l. Electron spin adds s=1/2; total angular momentum combines orbital and spin contributions.

## Apply the conditions

Electric-dipole selection rules include Δl=±1 and Δm=0,±1. These rules concern a specific transition mechanism; forbidden does not mean impossible by every mechanism.

## Check the conclusion

Fine structure, spin–orbit effects and Zeeman splitting 塞曼分裂 refine the simplest hydrogen picture. In many-electron atoms, shielding and electron interactions invalidate direct hydrogen-energy substitution.

## Worked method

For a nonrelativistic one-electron ion, neglecting fine structure,

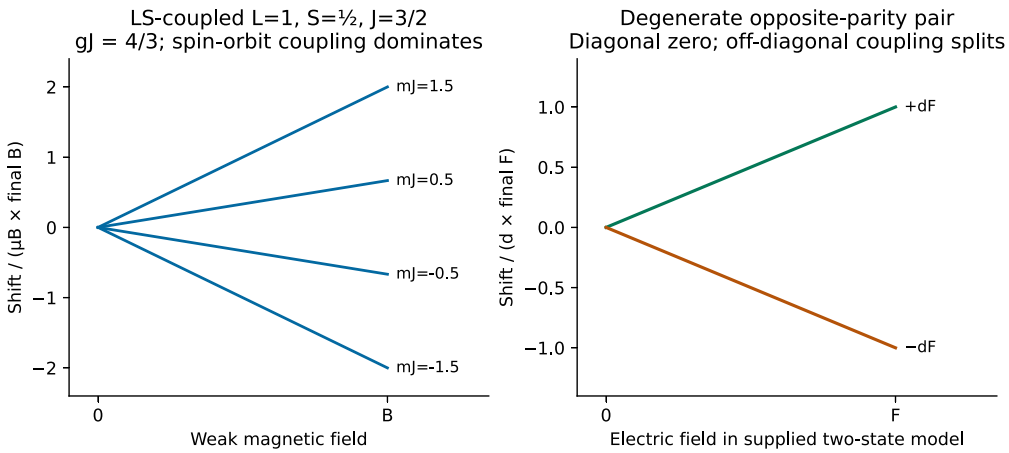
$$E_n = -13.6\,\text{eV}\,Z^2/n^2.$$

For Z = 2, an emission from n = 3 to n = 2 gives positive photon energy:

$$E_\gamma = E_3 - E_2 = (13.6\,\text{eV})(2^2)(1/2^2 - 1/3^2) = 7.56\,\text{eV}.$$

$$\lambda = hc/E_\gamma = (1240\,\text{eV nm})/(7.56\,\text{eV}) = 164\,\text{nm}.$$

An energy difference alone does not guarantee an electric-dipole transition; the participating orbital states must satisfy the selection rules.




*Weak-field Zeeman shifts are $gJ \cdot mJ \cdot \mu B \cdot B$; an electric field splits a degenerate opposite-parity pair linearly*

## Check conditions and vocabulary

Do not make the final-state energy minus initial-state energy negative for an emitted photon.

**selection rule** 选择定则: A restriction on transitions for a specified interaction mechanism.

**Zeeman splitting**: Magnetic-field splitting of atomic energy levels.



# Solid-state, nuclear and particle models

GRE Subject Test · GRE Physics

A semiconductor's conductivity depends on available states and carriers, not merely on the number of electrons in the material.

Prerequisites: 25, 45, 46.

- Use band and carrier descriptions
- Apply radioactive decay and binding-energy ideas
- Check conservation laws in particle reactions

## Choose the system and model

Filled bands cannot carry ordinary current without accessible nearby states. Metals have partially filled or overlapping bands; semiconductors have a small band gap 带隙. Donor and acceptor doping change carrier populations.

## Use the governing relation

Fermi–Dirac occupancy is 1/[exp((E−$\mu$)/(kT))+1]. At zero temperature it becomes a step at the Fermi energy; thermal broadening does not make every state half occupied.

## Apply the conditions

Radioactive populations obey N=N0e^(−$\lambda$t), activity $\lambda$N and half-life 半衰期 ln2/$\lambda$. Binding energy 结合能 comes from a mass deficit multiplied by $c^2$; compare binding energy per nucleon when discussing stability.

## Check the conclusion

Nuclear and particle processes must conserve charge, energy, momentum and the applicable quantum numbers. Beta decay includes a neutrino or antineutrino; a two-body electron-only story cannot explain its continuous energy spectrum.

## Worked method

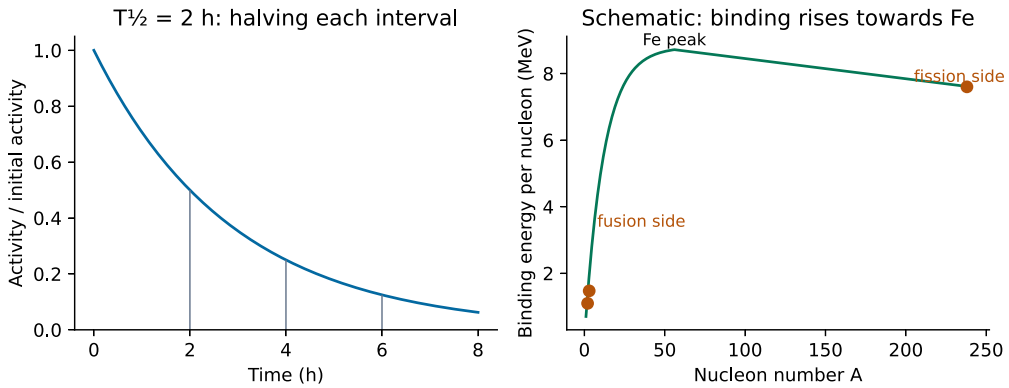
Binding energy is the positive energy required to separate a nucleus into free nucleons. For a reaction use total initial and final rest masses, with a consistent mass convention.

$$Q = (M_i - M_f)c^2.$$

A mass decrease of 0.0040 atomic mass units gives

$$Q = \Delta M c^2 = (0.0040\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = 3.73\,\mathrm{MeV}.$$

Positive Q means energy release. Charge, nucleon number, energy and momentum must still balance; Q alone does not establish a reaction rate.




*Each half-life halves the activity; energy is released by moving towards the iron peak, by fusion from the left or fission from the right*

## Check conditions and vocabulary

Activity and surviving population both decay, but their units differ: activity is transitions per second.

**band gap**: An energy interval with no allowed bulk electronic states in the band model.

**half-life**: Time for a radioactive population to halve.



# Hamiltonian mechanics and astrophysical scaling

GRE Subject Test · GRE Physics

Energy methods connect a spring oscillator and an orbit, but their coordinates and physical assumptions differ.

Prerequisites: 26, 41.

- Construct a simple Hamiltonian
- Use canonical equations
- Apply gravitational and radiative scaling

## Choose the system and model

The canonical momentum 正则动量 is p=∂L/∂qdot. The Legendre transform H=p qdot−L gives a Hamiltonian when velocities can be expressed using coordinates and momenta.

## Use the governing relation

Hamilton's equations are qdot=∂H/∂p and pdot=−∂H/∂q. For an ordinary oscillator $H=p^2/(2m)+kq^2/2$, these reproduce Newton's equation.

## Apply the conditions

For a circular gravitational orbit $v^2=GM/r$ and $period^2=4\pi^2r^3/(GM)$. The assumptions include a dominant central mass and a circular approximation; elliptical orbits use the semimajor axis in Kepler's law. For a circular satellite of mass m, angular momentum magnitude is mrv=m sqrt(GMr), so identical satellites have L proportional to sqrt(r). A radius ratio 9 therefore gives angular-momentum ratio 3, while the period ratio is 27. This distinction follows from the same centripetal-force relation; do not use period scaling for angular momentum.

## Check the conclusion

Luminosity 光度 and received flux obey $F=L/(4\pi d^2)$ for isotropic radiation. A blackbody has $L=4\pi R^2\sigma T^4$; its spectral peak shifts inversely with temperature. Distinguish intrinsic luminosity from observed brightness.

## Worked method

For a regular Lagrangian $L = m\dot{q}^2/2 - kq^2/2$, canonical momentum is

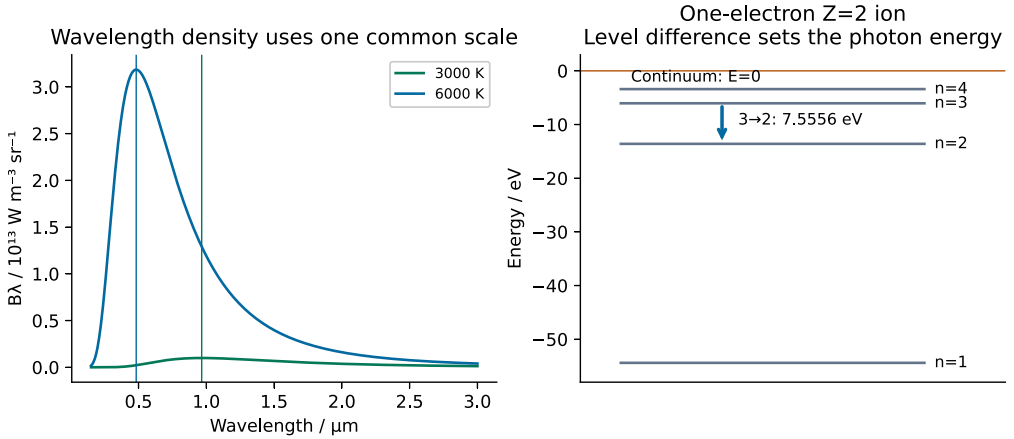
$$p = \partial L/\partial \dot{q} = m\dot{q}, \qquad \dot{q} = p/m.$$

Substitute the velocity into the Legendre transform:

$$H = p\dot{q} - L = p^2/(2m) + kq^2/2.$$

$$\dot{q} = \partial H/\partial p = p/m, \qquad \dot{p} = -\partial H/\partial q = -kq.$$

Combining these gives $\ddot{q} = -(k/m)q$. The inversion of momentum to velocity is required; it cannot be assumed for every constrained or singular Lagrangian.




*Doubling the temperature halves the peak wavelength; for a $Z = 2$ ion every level is $Z^2 = 4$ times the hydrogen value*

## Check conditions and vocabulary

A Hamiltonian equals total mechanical energy only under the relevant system assumptions; do not infer this universally from its name.

**canonical momentum**: The derivative of the Lagrangian with respect to a generalised velocity.

**luminosity**: Total emitted power, distinct from flux at an observer.



# Collisions, work and oscillator energy

GRE Subject Test · GRE Physics

Two carts approach from different directions. After they stick, their momentum can be conserved while their kinetic energy falls.

Prerequisites: 1.

- Compute vector momentum and energy loss in sticking collisions
- Compare fixed-force springs and use work–energy conditions
- Derive small-oscillation periods and oscillator energies

## Choose the system and model

Choose an isolated system during the short collision. With negligible external impulse 外力冲量, conserve total momentum separately in every Cartesian direction. If masses m1 and m2 stick, their common velocity is (m1 v1+m2 v2)/(m1+m2). Find the magnitude only after adding vectors. Initial kinetic energy is the sum of each ½m|v|²; final kinetic energy uses the total mass and the common speed. Their difference becomes internal energy or deformation, not missing momentum. A perfectly inelastic collision means sticking, not final rest.

## Use the governing relation

The work–energy theorem ΔK=∫F · dr uses net force along displacement. For a constant parallel force over distance s, ΔK=Fs. Speed doubling quadruples kinetic energy. Conservation of mechanical energy is a separate statement requiring no unaccounted nonconservative work. At a spring displacement x, U=½kx². Under the same applied force F, equilibrium extension is F/k and stored energy is F²/(2k): a stiffer spring stretches less and stores less energy. Under the same extension, it stores more. Identify which condition is held fixed.

## Apply the conditions

Near a stable equilibrium x0, expand U≈U(x0)+½U″(x0)(x−x0)²; the effective stiffness is positive U″(x0). The motion obeys m xddot=−k_eff(x−x0), with ω=sqrt(k_eff/m) and period 2π/ω. A simple pendulum obeys θddot+(g/L)sinθ=0. For small angles sinθ≈θ, so T=2πsqrt(L/g). This approximation explains period–length scaling; a large amplitude requires a correction, and the mass cancels for an ideal pendulum.

## Check the conclusion

For an undamped harmonic oscillator, total energy is ½m v²+½kx²=½kA². At equilibrium all its energy is kinetic; at a turning point 转折点 its speed is zero and all its energy is potential. Angular frequency is not ordinary frequency: ω=2πf. If the same oscillator has equilibrium speed vmax, A=vmax/ω. Distinguish the instant at which speed is measured from the release displacement, and keep SI units when a millijoule answer is requested.

## Worked method

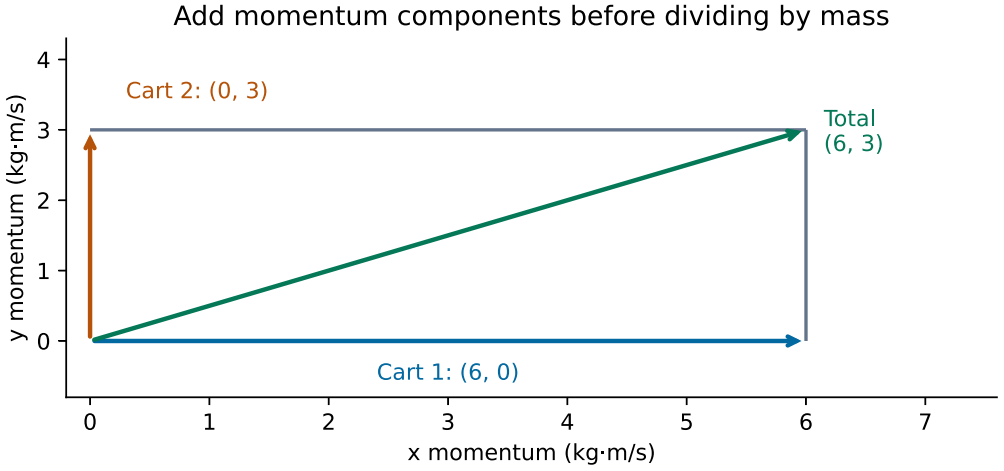
Two carts stick with negligible external impulse. Add momentum vectors 动量矢量, then divide by total mass. Known: m1 = 2 kg, v1 = (3,0) m/s; m2 = 1 kg, v2 = (0,3) m/s.

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2\,\mathrm{kg})(3,0)\,\mathrm{m/s} + (1\,\mathrm{kg})(0,3)\,\mathrm{m/s}}{3\,\mathrm{kg}} = (2,1)\,\mathrm{m/s}.$$

$$K_i = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(2\,\mathrm{kg})(3\,\mathrm{m/s})^2 + \tfrac{1}{2}(1\,\mathrm{kg})(3\,\mathrm{m/s})^2 = 13.5\,\mathrm{J}.$$

$$K_f = \tfrac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 = \tfrac{1}{2}(3\,\mathrm{kg})(5\,\mathrm{m^2/s^2}) = 7.5\,\mathrm{J}.$$

The 6 J difference becomes internal energy. Kinetic energy is not conserved in sticking.




*Add momentum as vectors first, then divide by the total mass*

## Check conditions and vocabulary

Do not add incoming speed magnitudes as momenta, or conserve kinetic energy in a sticking collision. Fixed-force and fixed-extension spring comparisons have opposite answers.

**external impulse**: Time integral of the net force from outside the chosen system.

**turning point**: An extreme oscillator position where its instantaneous speed is zero.

# Rotation, buoyancy and terminal-speed balances

GRE Subject Test · GRE Physics

Turning a floating block upside down can change how much wood is submerged even though the total weight stays the same.

Prerequisites: 10.

- Use pivot torque and rotational inertia consistently
- Partition rolling kinetic energy with the no-slip constraint 无滑动约束
- Balance buoyancy and drag for composite bodies

## Choose the system and model

Torque about a fixed pivot is r×F; its magnitude uses the perpendicular lever arm. For a uniform rod of length L pivoted at one end, gravity acts at its centre L/2 and the moment of inertia 转动惯量 is $ML^2/3$. If its angle $\theta$ is measured from vertical, gravitational torque magnitude is Mg(L/2)sin$\theta$, so angular-acceleration magnitude is 3g sin$\theta$/(2L). Define the positive rotation direction before assigning a sign. Using the centre-of-mass inertia $ML^2/12$ without the parallel-axis correction gives a wrong acceleration.

## Use the governing relation

An object rolling without slipping satisfies v=R$\omega$. Write total kinetic energy as ½Mv²+½I$\omega^2$. With I=$\beta$MR², the rotational fraction is $\beta/(1+\beta)$; for a solid disk $\beta$=1/2 and the fraction is 1/3. A hoop has $\beta$=1 and fraction 1/2. Static friction can supply the torque required for rolling without dissipating energy at an instantaneously stationary contact on a fixed surface. A sliding object does not satisfy the no-slip relation automatically.

## Apply the conditions

For a floating composite, buoyancy equals the weight of displaced fluid, summed over every submerged part. Let a wood block have volume V and a dense stone volume Vs. If the stone is above the water, only submerged wood contributes: $\rho$water g fV equals total weight. If the stone is attached below and fully submerged, its displaced volume reduces the wood's required submerged fraction to f=Vs/V. The stone does not cease to weigh anything; the water now supplies some of its support. Check whether the assumed orientation and flotation are physically possible.

## Check the conclusion

Terminal speed 终端速度 means zero acceleration, not zero speed or zero force. With quadratic drag C v² and a downward weight Mg, neglecting buoyancy gives vt=sqrt(Mg/C). Two otherwise identical balls share the drag coefficient C; doubling mass then multiplies terminal speed by sqrt(2). If buoyancy matters, replace Mg by (M−$\rho$fluid V)g. A linear-drag model instead gives a different mass scaling. State the specified drag law and compare force balances, rather than transferring a formula between models.

## Worked method

A body rolls without slipping on a fixed surface. The no-slip constraint is v = R omega.

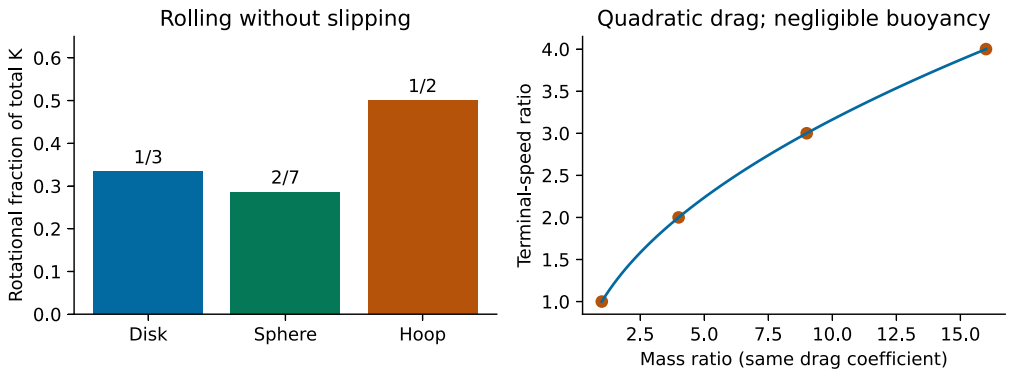
$$K = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}Mv^2(1+\beta), \qquad I = \beta MR^2.$$

For a solid disk, beta = 1/2, so the rotational share is

$$\frac{K_{rot}}{K} = \frac{\beta}{1+\beta} = \frac{1/2}{3/2} = \frac{1}{3}.$$

For a uniform rod pivoted at one end, use $I = ML^2/3$, not its centre-of-mass inertia. About this pivot $\tau = -Mg(L/2)\sin\theta$, for theta measured from downward vertical.

$$\ddot{\theta} = \tau/I = -3g\sin\theta/(2L).$$




*Rolling shares energy by $I/(mr^2)$; quadratic drag gives terminal speed $\propto \sqrt{m}$*

## Check conditions and vocabulary

Use the inertia about the chosen pivot. In flotation, include the stone's displaced volume as well as its weight. Terminal speed requires a specified drag law.

**moment of inertia**: Mass-weighted squared perpendicular distance from a specified rotation axis.

**terminal speed**: Steady speed at which opposing forces balance.



# Coupled acceleration and relative motion

GRE Subject Test · GRE Physics

## Coupled acceleration 加速度 and relative motion

A load released from an aircraft keeps the aircraft's horizontal velocity. Relative to the aircraft, its initial motion can therefore be purely downward.

Prerequisites: 1, 11.

- Solve coupled-body equations using one acceleration constraint 约束
- Transform velocities between inertial frames
- Check force and energy limits for constrained motion

## Choose the system and model

For a mass m1 on a frictionless horizontal table connected over an ideal massless pulley to a hanging mass m2, choose table motion and downward hanging motion as positive. A taut inextensible string gives one acceleration magnitude. Write T=m1a and m2g−T=m2a. Adding eliminates the internal tension, so a=m2g/(m1+m2) and T=m1m2g/(m1+m2). Thus T<m2g when both masses are positive. Setting tension equal to hanging weight silently assumes zero acceleration.

## Use the governing relation

The common tension assumption requires an ideal massless string and pulley with negligible friction and inertia. A pulley with rotational inertia can have different tensions, with (T2−T1)R=I$\alpha$ and a=R$\alpha$ if the string does not slip. Draw each body separately: a force internal to the combined system remains an external force on one chosen body. Constraint forces can cancel from a system equation while still being needed for individual motion.

## Apply the conditions

For two inertial frames with constant relative velocity 相对速度 V, Galilean velocity transformation is v_relative=v_ground−V. A payload released by a level aircraft initially has the aircraft's horizontal speed. With negligible air resistance, horizontal ground velocity remains constant and vertical velocity becomes −gt if upward is positive. Relative to that aircraft, horizontal velocity is zero and downward speed is gt. Position is a different question: vertical displacement is $-\frac{1}{2}gt^2$. Do not confuse ground speed with relative speed.

## Check the conclusion

Check limiting cases after solving. As m1 approaches zero, the ideal coupled acceleration approaches g and tension approaches zero; as m1 grows very large, acceleration approaches zero and tension approaches m2g from below. An energy derivation gives m2g s=½(m1+m2)v² for release from rest and an ideal pulley, agreeing with v²=2as. This agreement checks the equations; it does not permit energy conservation if friction or other unaccounted work is present.

## Worked method

A table mass m1 and hanging mass m2 share an acceleration through a taut ideal string. Choose rightward table motion and downward hanging motion positive.

$$T = m_1 a, \qquad m_2 g - T = m_2 a.$$

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(2\,\mathrm{kg})(10\,\mathrm{m/s^2})}{3\,\mathrm{kg} + 2\,\mathrm{kg}} = 4\,\mathrm{m/s^2}.$$

$$T = m_1 a = (3\,\mathrm{kg})(4\,\mathrm{m/s^2}) = 12\,\mathrm{N}.$$

T is less than the hanging weight 20 N. A nonzero net downward force is needed for acceleration.

Subtract aircraft velocity; components in m/s

| Velocity after 3 s | Horizontal | Vertical |
|---|---|---|
| Ground | 80 | −30 |
| Aircraft | 80 | 0 |
| Payload relative to aircraft | 0 | −30 |

*Subtract the aircraft's velocity component by component*

## Check conditions and vocabulary

A massless ideal pulley equates tension, not tension and weight. Subtract frame velocities component by component before taking a speed magnitude.

**constraint**: A condition linking allowed positions or motions of connected bodies.

**relative velocity**: Velocity of one object measured in another moving frame.

# Electrostatic superposition, flux and conductors

GRE Subject Test · GRE Physics

A charged conductor can have zero field inside its material while its potential remains nonzero.

Prerequisites: 2, 47.

- Add Coulomb forces as vectors using the actual geometry
- Compute flux from symmetry or solid angle 立体角 with a stated orientation
- Find conductor fields and potentials using induced surface charges

## Choose the system and model

For a point charge Q at position r0, E(r)=Q(r−r0)/(4πε0|r−r0|$^3$). Superpose vectors, not field magnitudes. For two positive charges attracting an electron, forces in the same direction add; opposite directions subtract; perpendicular components combine by Pythagoras. Define the observation point and the direction of each force first. Potential is the scalar sum V=ΣQ/(4πε0r) with zero at infinity. A zero potential does not generally mean a zero field, since E=−∇V.

## Use the governing relation

Gauss's law is the closed-surface integral $\oint$E · dA=Q_enclosed/ε0. It determines a local field simply only when symmetry makes the normal field uniform or makes other flux contributions zero. A charge near an infinite plane sends half its total flux through that plane in magnitude: the plane subtends solid angle 2π out of 4π. The result is independent of distance and lateral position, but its sign depends on the chosen plane normal and charge sign. An open plane does not enclose a charge; using closed-surface wording for it is incorrect.

## Apply the conditions

In electrostatic equilibrium 静电平衡, the electric field inside conducting material is zero and the conductor has constant potential. Place a charge Q at the centre of a conducting spherical shell with inner radius a, outer radius b and shell net charge q. A Gaussian surface inside the material requires inner-surface charge −Q, so the outer surface has q+Q. Spherical symmetry makes the external field that of total Q+q at the centre. The material's potential, zero at infinity, is (Q+q)/(4πε0b), independent of the material observation radius.

## Check the conclusion

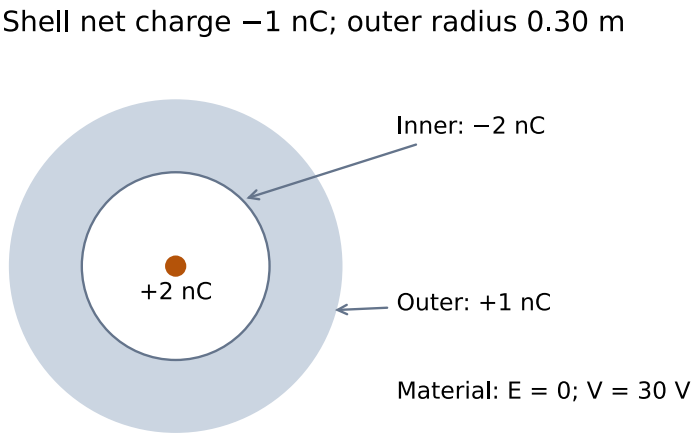
In the cavity of that centred-charge shell, the field is Q/(4πε0r$^2$) radially for 0<r<a. Potential includes both the point-charge contribution and constant shell contributions; continuity holds across each surface even though the normal field jumps at a surface charge. Keep cavity, conductor material and exterior separate. An off-centre charge still induces total inner charge −Q, but its cavity field is no longer the simple centred radial field. Zero interior conductor field follows equilibrium, not an assumption that every surface charge distribution is uniform.

## Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q. Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{inner} = -Q$. Charge conservation gives $Q_{outer} = q + Q$. With outer radius b and zero potential at infinity,

$$V_{metal} = \frac{q+Q}{4\pi\epsilon_0 b} = \frac{(9.0\times10^9\,\mathrm{N\,m^2/C^2})(1.0\times10^{-9}\,\mathrm{C})}{0.30\,\mathrm{m}} = 30\,\mathrm{V}.$$

Here Q = +2 nC and q = -1 nC. The cavity is not conducting material; its field need not vanish.




*Induced charges make E = 0 in the metal; the shell's potential is k(+1 nC)/0.30 m = 30 V*

## Check conditions and vocabulary

Do not confuse the open-plane half-flux result with enclosed charge. A conductor's zero field fixes a constant potential, not necessarily zero potential.

**solid angle**: Angular area subtended by a surface, measured in steradians.

**induced surface charge** 感应表面电荷: Charge rearranged on a conductor to satisfy electrostatic equilibrium.



# Circuit power, induction and charged-particle motion

GRE Subject Test · GRE Physics

A resistor in a parallel branch sees the branch voltage, which can differ from the battery voltage because another resistor is in series.

Prerequisites: 2, 30.

- Solve resistor networks and distinguish rms from peak AC quantities
- Apply flux linkage 磁通链, Lenz direction and the current-loop field
- Use charge-to-mass ratios and balance crossed-field forces

## Choose the system and model

For steady ideal resistor circuits, series resistances add and parallel conductances add. Reduce a network 电路网络 while preserving which nodes share the same voltage. With a 24 V source, a 2 Ω series resistor and parallel 3 Ω and 6 Ω branches, the parallel equivalent is 2 Ω. Total current is 6 A, branch voltage is 12 V and branch currents are 4 A and 2 A. The current through every component is not necessarily the total current; verify Kirchhoff current and voltage sums after reduction.

## Use the governing relation

For sinusoidal current I=Imax sin(ωt−φ), Irms=Imax/sqrt(2). A series RLC circuit has impedance magnitude sqrt($R^2$+(ωL−1/(ωC))$^2$) and average dissipated power $I_{rms}^2$R=Vrms Irms cosφ. Ideal inductors/capacitors exchange stored energy with zero average dissipation. If rms current is already given, do not insert another factor 1/2. An ideal rectifying diode conducts in its forward direction and blocks in reverse: a suitably oriented series diode clips the negative half of a sinusoidal resistor voltage; it does not create a full-wave rectifier by itself.

## Apply the conditions

Magnetic flux is $\int B \cdot dA$, or BA cosθ for a uniform field with θ measured from the area normal. Faraday emf is −N dΦ/dt. For changing perpendicular field and fixed loop geometry, its average magnitude is NA|ΔB|/Δt. Determine the sign or current direction using opposition to the flux change, not opposition to the field itself. The magnetic field at the centre of one circular loop is μ0I/(2R), obtained from Biot–Savart: every element contributes in the same axial direction. An N-turn compact coil multiplies this result by N.

## Check the conclusion

For a nonrelativistic charge with velocity perpendicular to uniform B, magnetic force |q|vB provides centripetal force $mv^2/r$. Thus r=mv/(|q|B) and cyclotron angular frequency is |q|B/m, independent of speed in this approximation. Charge sign sets rotation direction. A deuteron has approximately twice proton mass and the same charge, while an alpha particle has four times its mass and twice its charge; their frequencies are each half the proton value. In crossed E and B fields, undeflected motion requires electric and magnetic forces opposite and v=E/B. Check vector directions before using this magnitude.

## Worked method

A 24 V ideal source drives 2 ohms in series with parallel 3 and 6 ohm branches. Identify nodes before reducing the network.

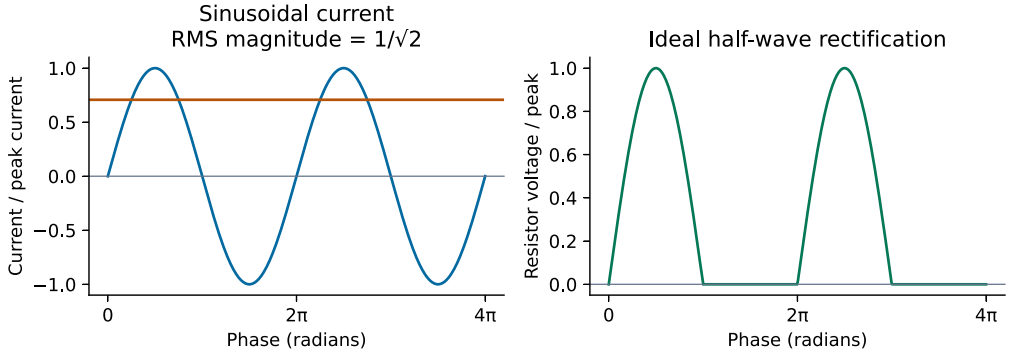
$$R_p = (1/R_1 + 1/R_2)^{-1} = (1/(3\,\Omega) + 1/(6\,\Omega))^{-1} = 2\,\Omega.$$

$$I = V/(R_s + R_p) = (24\,\mathrm{V})/(4\,\Omega) = 6\,\mathrm{A}.$$

$$V_p = IR_p = (6\,\mathrm{A})(2\,\Omega) = 12\,\mathrm{V}.$$

$$I_2 = V_p/R_2 = (12\,\mathrm{V})/(6\,\Omega) = 2\,\mathrm{A}.$$

The other branch carries 4 A; the branch currents sum to the source current.




*RMS of a sine is peak/√2; a half-wave rectifier passes only the positive half-cycles*

## Check conditions and vocabulary

Do not apply battery voltage to every branch or halve an rms-current power again. Flux angle uses the area normal; a selector also needs opposing force directions.

**root mean square** 均方根: Square root of the mean squared value, used for effective AC current or voltage.

**flux linkage**: Sum of magnetic flux through a coil's turns.



# Maxwell waves, energy flow and boundary conditions

GRE Subject Test · GRE Physics

A charged capacitor can change its electric field across a gap even though no conduction current crosses the gap.

Prerequisites: 13, 29, 47.

- Use displacement current 位移电流 and material wave-speed relations
- Determine wave-field directions and radiated energy flow
- Apply Maxwell boundary conditions without confusing field components

## Choose the system and model

Ampere–Maxwell law in vacuum is $\oint B \cdot dl = \mu 0(I_conduction + \varepsilon 0\, d\Phi E/dt)$. The added displacement-current term depends on the time rate of electric flux through the chosen surface. It ensures consistent results for a loop whose spanning surface either crosses a capacitor wire or passes between the plates. Displacement current is not magnetic flux or an integral over earlier electric flux. For a uniform plate field, the gap contribution is $\varepsilon 0 A\, dE/dt$; signs follow the oriented surface.

## Use the governing relation

In a homogeneous, linear, lossless dielectric 电介质 with permeability $\mu$ and permittivity $\varepsilon$, Maxwell equations give wave speed $v = 1/\mathrm{sqrt}(\mu\varepsilon)$ and refractive index $n = c/v = \mathrm{sqrt}(\mu_r \varepsilon_r)$. For nonmagnetic material $\mu_r \approx 1$, so $v = c/\mathrm{sqrt}(\varepsilon_r)$. Use the stated frequency-dependent material constants when dispersion matters; a static dielectric constant need not describe every optical frequency. Vacuum waves have E/B=c; in this simple medium the relation is E/B=v.

## Apply the conditions

For a plane wave travelling along unit vector n, B=(n×E)/v. Both fields are perpendicular to propagation and to each other. Phase kz−ωt propagates toward +z, while kz+ωt propagates toward −z. If E is along x+y and propagation is +z, B is along −x+y, because z×x=y and z×y=−x. The Poynting vector 坡印廷矢量 S=E×H (E×B/$\mu$ in this medium) points along energy transport. In the radiation zone of an accelerating charge, energy flux is outward from the source, not necessarily in the instantaneous direction of charge motion.

## Check the conclusion

Maxwell's divergence equation $\nabla \cdot \mathrm{B}{=}0$ gives continuity of the normal B component across an interface: a thin pillbox has no magnetic charge inside. The tangential H jump is related to surface current; it need not vanish. Under an ideal superconducting Meissner-state model with zero interior B, the exterior normal component at the boundary must therefore be zero, leaving any nonzero exterior B tangent to the surface. This conclusion is conditional on the stated zero-interior model; it does not follow by assuming all exterior field is zero. For electrostatics, normal D can instead jump by free surface charge.

## Worked method

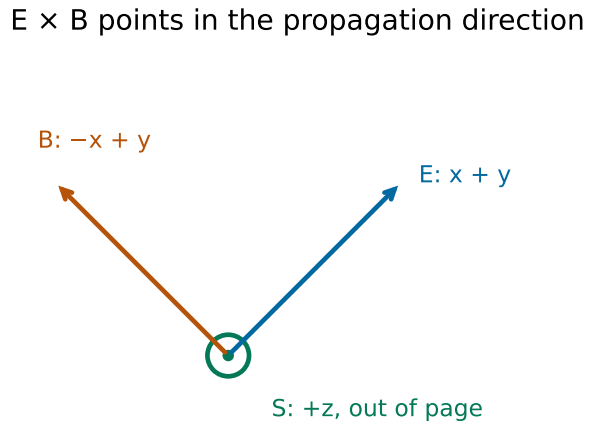
A plane wave travels along positive z in a linear lossless nonmagnetic dielectric. With relative permittivity 9,

$$v = c/\sqrt{\epsilon_r} = (3.00 \times 10^8\,\mathrm{m/s})/3 = 1.00 \times 10^8\,\mathrm{m/s}.$$

If E points along positive x, $\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{E}/v$ points along positive y. For E0 = 200 V/m,

$$B_0 = E_0/v = (200\,\mathrm{V/m})/(1.00 \times 10^8\,\mathrm{m/s}) = 2.00\,\mu\mathrm{T}.$$

$\mathbf{E} \times \mathbf{H}$ points along propagation. Check that material constants apply at the wave frequency.




*$E \times B$ gives the direction in which the wave travels*

## Check conditions and vocabulary

Displacement current uses changing electric flux. Zero interior B fixes the exterior normal component, not every exterior component. Distinguish wave propagation from particle motion.

**displacement current**: Electric-flux time-derivative contribution in Ampere–Maxwell law.

**Poynting vector**: Electromagnetic energy flux per unit area and time.

# Acoustic Doppler echoes and standing-wave boundaries

GRE Subject Test · GRE Physics

A driver hears an echo from a stationary wall at a shifted pitch: the outbound and returning sound each acquire a Doppler factor 多普勒因子.

Prerequisites: 3.

- Apply source and observer Doppler factors in the medium frame
- Derive allowed pipe frequencies from displacement boundary conditions
- Distinguish frequency, wavelength and boundary changes

## Choose the system and model

For sound in a stationary medium, separate source motion from observer motion. A source approaching at speed u_s compresses wavefront spacing and produces received frequency f c/(c−u_s) at a stationary observer. An observer approaching a stationary source at speed u_o meets more wavefronts per second and measures f(c+u_o)/c. Speeds are measured relative to the medium; the acoustic source and observer formulas are not symmetric under exchanging roles. Receding motion reverses the appropriate sign. These expressions assume subsonic motion along the propagation line.

## Use the governing relation

For a siren moving toward a stationary reflecting wall with speed u, the wall first receives f_wall=f c/(c−u). Reflection from a stationary wall preserves frequency in the medium frame. The moving driver then approaches the returning wavefronts and receives f_echo=f_wall(c+u)/c=f(c+u)/(c−u). At small u/c the fractional shift is approximately 2u/c, but the exact expression has different numerator and denominator. A moving reflecting surface needs its own Doppler step; do not reuse the stationary-wall result blindly.

## Apply the conditions

At an ideal open pipe end, air displacement is an antinode and pressure variation is a node. At a rigid closed end, displacement is a node and pressure is an antinode. For both ends open, length L contains n half-wavelengths, giving f_n=nc/(2L), n=1,2,… . For one end closed, it contains an odd number of quarter-wavelengths, giving f_n=(2n−1)c/(4L). Real pipes may need end corrections; the ideal GRE model uses the stated length without inventing an adjustment.



## Check the conclusion

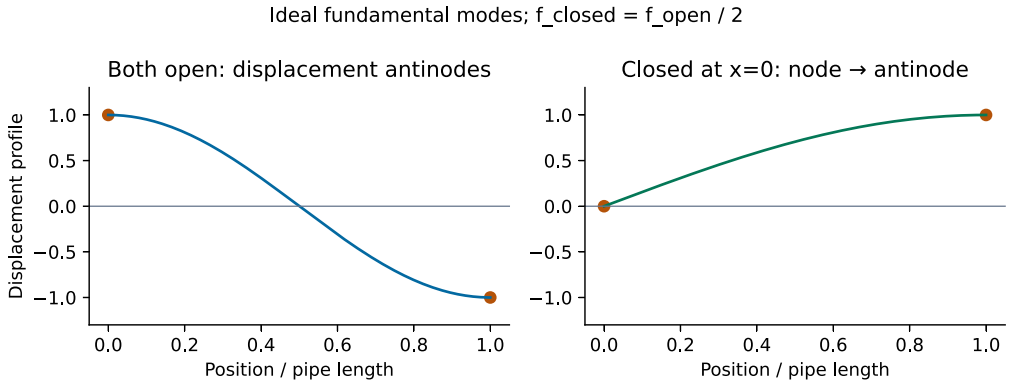
Closing one end of a previously both-open pipe halves its fundamental and leaves only odd multiples of that new fundamental. Its old frequencies were integer multiples of c/(2L), which are even multiples of c/(4L), so none is an allowed frequency of the new ideal one-closed spectrum. This differs from simply deleting even harmonics while retaining the old fundamental. When the medium is unchanged, wave speed is unchanged; frequency and wavelength change together to satisfy the new boundary geometry.

## Worked method

A siren approaches a stationary wall in still air. Treat outward and return journeys separately. The wall first receives $f_w = fc/(c-u)$; reflection keeps this frequency in the medium frame. The moving listener then receives $f_e = f_w(c+u)/c$.

$$f_e = f\frac{c+u}{c-u} = (600\,\text{Hz})\frac{340\,\text{m/s}+20\,\text{m/s}}{340\,\text{m/s}-20\,\text{m/s}} = 675\,\text{Hz}.$$

This Doppler effect 多普勒效应 uses velocities relative to the medium. It is not the light-wave formula.




*For the same length, the closed pipe's fundamental is half the open pipe's*

## Check conditions and vocabulary

An echo needs both Doppler factors. Closing a pipe changes the fundamental; odd harmonics are counted from the new fundamental, not the old one.

**displacement antinode** 位移波腹: A standing-wave position where the air displacement amplitude is maximal.

**Doppler factor**: Frequency multiplier produced by a specified source or observer motion.

# Optical instruments, path differences and refraction

GRE Subject Test · GRE Physics

A gas cell in one arm of a Michelson interferometer 干涉仪 is crossed twice. Counting only one passage halves the inferred refractive-index change.

Prerequisites: 3, 33.

- Calculate telescope magnification and grating resolving power 分辨本领
- Relate Michelson fringe counts to double-pass optical path changes
- Trace surface normals before applying Snell and reflection laws

## Choose the system and model

For an ideal astronomical telescope adjusted for relaxed viewing at infinity, the objective forms its focal-plane image and the eyepiece collimates the emerging light. Lens separation is f_objective+f_eyepiece; angular magnification magnitude is f_objective/f_eyepiece. The usual two-converging-lens telescope produces an inverted image, represented by a negative signed angular magnification under a consistent convention. A finite final-image distance changes lens separation; do not use the infinity adjustment without checking the condition.

## Use the governing relation

Resolving power R=$\lambda/\Delta\lambda$ describes the smallest distinguishable wavelength separation near $\lambda$. For an ideal diffraction grating in order m with N illuminated slits, R=mN. This is a resolution criterion, not simply the angular position formula d sin$\theta$=m$\lambda$. Increasing illuminated slit count sharpens the principal peaks; increasing slit spacing mainly changes their angular locations. A measured $\lambda$=600 nm and $\Delta\lambda$=3 nm gives R=200, independent of converting both lengths to metres because their ratio uses matching units.

## Apply the conditions

Michelson interference depends on the optical path difference between the arms. If one arm contains a gas cell of geometric length L and index changes from n to 1 upon evacuation, a double passage changes path by 2L(n−1). If N fringes pass, the magnitude of this change is N$\lambda$, so n−1=N$\lambda$/(2L). For a moving mirror instead, displacement $\Delta$x gives path change 2$\Delta$x. Fringes count phase cycles; one fringe is one wavelength of optical path, not one wavelength of mirror motion.

## Check the conclusion

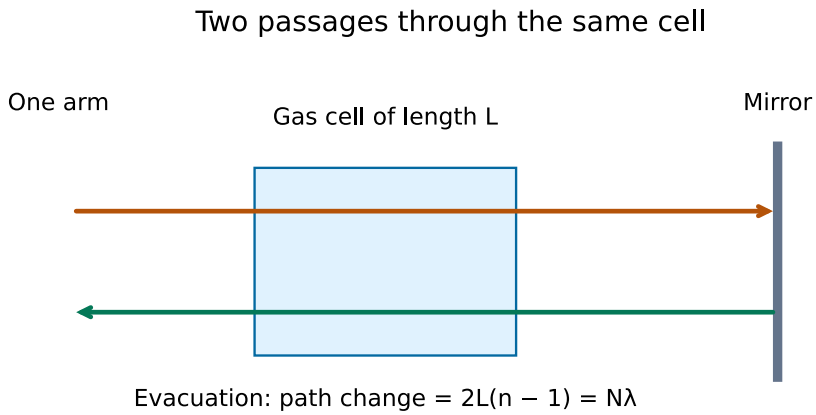
Snell's law n1 sin$\theta$1=n2 sin$\theta$2 uses angles from the local surface normal. Reflection has equal incoming and outgoing angles from that same normal. In a rectangular block, normals of neighbouring faces are perpendicular: an incidence angle measured from one normal is complementary to the angle of that same ray from the other. Draw the ray and normals before substitution. For incidence from index n into air, total internal reflection occurs only when sin$\theta$>1/n; equality is the critical grazing case. A stated partly refracted ray must satisfy the transmission condition.

## Worked method

A Michelson interferometer sends light twice through a gas cell of length L. Evacuation changes path by $2L(n-1)$. N fringes correspond to path change $N\lambda$.

$$n-1=\frac{N\lambda}{2L}=\frac{(120)(500\times10^{-9}\,\mathrm{m})}{2(0.10\,\mathrm{m})}=3.0\times10^{-4}.$$

Thus n = 1.00030. Moving a mirror is also double-pass; a fringe is not one wavelength of mirror motion.




*Evacuating the cell changes the optical path by $2L(n-1)=N\lambda$*

## Check conditions and vocabulary

Use optical path, not just geometric distance. Refraction angles are measured from the normal of the actual surface, and telescope focal-length addition assumes a final image at infinity.

**resolving power**: Wavelength divided by the smallest resolvable wavelength separation.

**optical path length** 光程: Geometric path weighted by refractive index along the ray.

# Fourier symmetry and wave superposition

GRE Subject Test · GRE Physics

A waveform can contain many harmonics while having no sine terms at all. Reflection symmetry tells you which coefficients vanish before integration.

Prerequisites: 3, 47.

- Use orthogonality 正交性 to identify Fourier coefficients
- Exploit even and odd parity without discarding the constant term
- Relate component amplitudes to interference and physical waveforms

## Choose the system and model

For a sufficiently regular real $2\pi$-periodic function, write f(x)=a0/2+Σ[a_n cos(nx)+b_n sin(nx)]. The coefficients are a_n=(1/π)∫from−πtoπ f(x)cos(nx)dx and b_n=(1/π)∫from−πtoπ f(x)sin(nx)dx, with a0 using n=0. Cosines and sines are orthogonal on a full period. The mean is a0/2, not a0; retain the constant term even when all nonzero-frequency coefficients of one family vanish.

## Use the governing relation

If f is even about the chosen origin, f(x)sin(nx) is odd and its symmetric integral is zero, so all b_n vanish. If f is odd, its mean and all a_n vanish. Symmetry depends on the origin: shifting the same physical signal can mix sine and cosine coefficients while leaving its harmonic frequencies unchanged. A nonnegative triangular waveform symmetric about x=0 can therefore have cosine harmonics and a nonzero mean without any sine harmonics.

## Apply the conditions

For the $2\pi$-periodic extension of f(x)=|x| on $[-\pi,\pi]$, the mean is $\pi/2$. Integrating x cos(nx) by parts on $[0,\pi]$ gives a_n=2[(-1)^n−1]/(πn²), so even-n cosine coefficients are zero and odd-n coefficients are $-4/(\pi n^2)$. All sine coefficients are zero. The $1/n^2$ decay reflects a continuous function with a slope discontinuity. A jump discontinuity often gives slower coefficient decay and partial-sum overshoot; do not infer the same convergence behaviour for every waveform.

## Check the conclusion

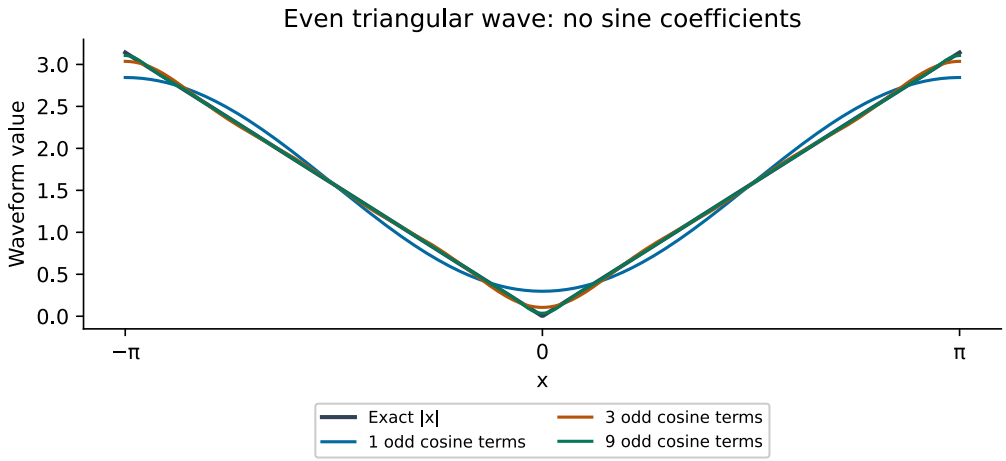
A Fourier sum adds amplitudes with their phases. For equal coherent monochromatic amplitudes A meeting with phase difference $\varphi$, resultant squared amplitude is $2A^2(1+\cos\varphi)$, so intensity is 2I0(1+cosφ). It is 4I0 in phase and zero at $\varphi=\pi$. Incoherent averaging removes the cross term, yielding 2I0. Harmonics at different frequencies can construct a shape over time; their instantaneous sum is not the sum of their individual intensities. Identify whether the question concerns a waveform, time-average power or coherent interference.

## Worked method

For a real periodic function, use $f = a_0/2 + \sum(a_n \cos nx + b_n \sin nx)$. Even symmetry 偶对称 makes all sine coefficients zero because the integrand is odd. For $f(x) = |x|$ on $[-\pi, \pi]$,

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,dx = \frac{2[(-1)^n - 1]}{\pi n^2}.$$

Integration by parts supplies the second expression. The mean is $a_0/2 = \pi/2$; only odd cosine harmonics survive. A cusp does not justify dropping the constant term.




*An even function needs only cosine terms; more terms sharpen the corner*

## Check conditions and vocabulary

Even symmetry removes sine coefficients, not the constant term or every cosine harmonic. Check the symmetry origin and whether fields are coherent before adding intensities.

**orthogonality**: A zero integral of the product of distinct basis functions over the specified interval.

**Fourier coefficient** 傅里叶系数: Weight of a sine, cosine or constant basis component in a periodic expansion.

# Gas processes, entropy and reversible cycles

GRE Subject Test · GRE Physics

A refrigerator 制冷机 can move more heat than the work supplied to it. That ratio is a coefficient of performance 性能系数, not an efficiency that must be below one.

Prerequisites: 4.

- Apply first-law work and heat signs to gas processes
- Compare reversible entropy balances and P–V cycle areas
- Distinguish engine efficiency from refrigerator coefficient of performance

## Choose the system and model

Use ΔU=Q−W, where Q is heat entering the system and W is work done by it. Quasistatic boundary work is ∫P dV, positive during expansion. For a fixed amount of ideal gas, internal energy depends only on temperature. At constant volume W=0, so ΔU=Q; for an isothermal ideal-gas process ΔU=0, so Q=W. Adiabatic means Q=0, not constant temperature. Over a cycle, the state returns and ΔU=0, so net Q equals net W even though individual legs have different heat transfers.

## Use the governing relation

For reversible isothermal expansion of an ideal gas, PV is constant. With constant heat capacities, a reversible adiabatic path obeys PV^γ=constant, with γ=Cp/Cv>1; through the same initial state its pressure falls faster as volume increases. An isobaric path is horizontal on a P–V plot. A clockwise closed cycle has positive ∮P dV; reversing direction changes the sign. For a rectangular loop the magnitude is ΔPΔV, and a triangular loop has half the corresponding bounding-rectangle area. Use Pa and $m^3$ for joules, not an unconverted litre value.

## Apply the conditions

For reversible heat transfer, dS=δQ_rev/T; entropy is a state function while heat is path dependent. A reversible system-plus-environment process has zero total entropy change, but the system's entropy alone can increase or decrease. Irreversible spontaneous processes produce nonnegative total entropy. A reversible isothermal expansion increases the gas entropy by nR ln(V2/V1); the reservoir loses the same amount. Reversibility does not require constant system temperature, zero work or zero internal-energy change.

## Check the conclusion

A reversible engine between TH and TC has efficiency W/QH=1−TC/TH. A reversible refrigerator instead has COP=QC/W=TC/(TH−TC), and a heat pump has QH/W=TH/(TH−TC). Use absolute kelvin temperatures. On a reversible T–S diagram, heat magnitude along an isotherm is T times the entropy change; the Carnot rectangle's area is the work magnitude. Refrigeration moves heat from cold to hot by consuming work, reversing the engine cycle. COP can exceed one without violating conservation because the moved heat is not supplied solely by the work.

## Worked method

A reversible refrigerator transfers entropy Delta S from Tc to Th.

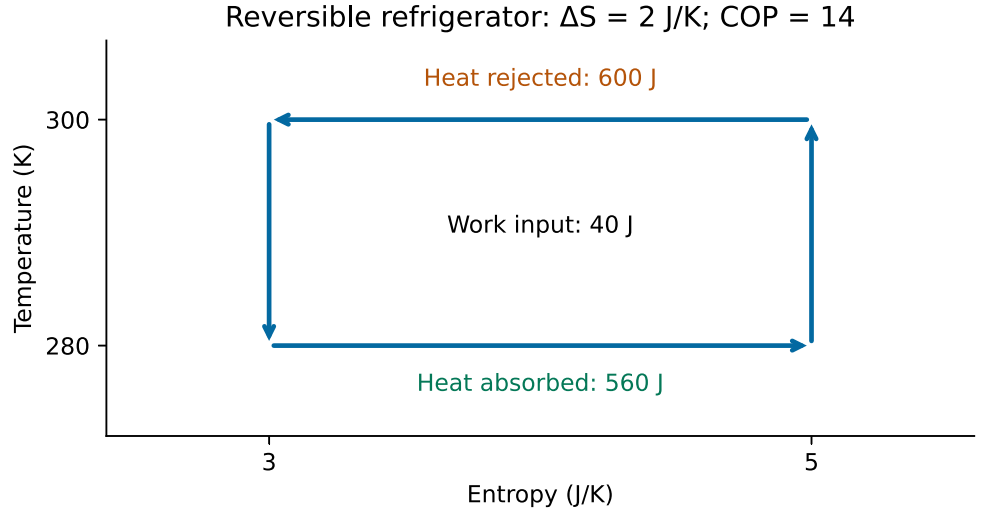
$$Q_c = T_c \Delta S = (270\,\mathrm{K})(3.0\,\mathrm{J/K}) = 810\,\mathrm{J}.$$

$$Q_h = T_h \Delta S = (300\,\mathrm{K})(3.0\,\mathrm{J/K}) = 900\,\mathrm{J}.$$

$$W_{in} = Q_h - Q_c = 900\,\mathrm{J} - 810\,\mathrm{J} = 90\,\mathrm{J}.$$

$$\mathrm{COP} = Q_c / W_{in} = (810\,\mathrm{J})/(90\,\mathrm{J}) = 9.$$

COP above one is possible because heat is moved, rather than all produced from work.




*For a reversible refrigerator, COP = 560/40 = 14*

## Check conditions and vocabulary

Zero total entropy production 熵产生 does not mean zero system entropy change. A refrigerator uses TC/(TH−TC), not the engine efficiency formula.

**entropy production**: Nonnegative total entropy generated by irreversibility.

**coefficient of performance**: Useful heat transferred divided by work input for a refrigerator or heat pump.



# Speed distributions and statistical ensembles

GRE Subject Test · GRE Physics

A gas can have zero mean velocity while its particles have substantial average speed. Opposite directions cancel in a vector mean; speed has no sign.

Prerequisites: 4, 35.

- Normalise continuous speed densities and interpret their units
- Distinguish vector means, speed means and most probable speeds
- Use Boltzmann weights with degeneracy and a partition function 配分函数

## Choose the system and model

For a continuous speed probability density 概率密度 f(v), f(v)dv approximates probability in a narrow interval and ∫from 0to∞ f(v)dv=1. The density has inverse-speed units and is not itself a probability. If P(v)dv counts particles in that interval, its area is the particle count N and f=P/N. A trapezoid rising over width v0, remaining flat over width 2v0, then falling over width v0 has area 3av0 for height a. Normalise by area before computing a mean or interpreting a plotted height.

## Use the governing relation

In an isotropic equilibrium gas without bulk flow, each velocity-component distribution is symmetric, so ⟨v_vector⟩=0. Speed v=|v_vector| is nonnegative and has positive mean. For a classical ideal gas, the Maxwell speed density is proportional to $v^2\exp[-mv^2/(2kBT)]$. Its mode is sqrt(2kBT/m), its mean is sqrt(8kBT/(πm)) and its rms speed is sqrt(3kBT/m). These are three different quantities. The $v^2$ factor makes speed density zero at v=0, even though the velocity-vector density is largest at the zero vector.

## Apply the conditions

For a continuous distribution, the probability of exactly one specified speed is zero; nonzero probabilities refer to intervals. This statement does not imply there are no particles with arbitrarily small speeds, or that a finite-resolution detector cannot record a zero bin. Temperature changes the scale of the Maxwell distribution: characteristic speeds are proportional to sqrt(T/m). Doubling temperature does not double the speed, and heavier particles are slower on average at the same temperature.

## Check the conclusion

In a canonical ensemble 正则系综 at temperature T, a state of energy E_i has weight exp(−E_i/(kBT)). Divide by partition function Z=Σexp(−E_i/(kBT)) to obtain probabilities. If an energy level has degeneracy g_i, its level probability is g_i exp(−E_i/(kBT))/Z. Equal energy per state does not imply equal probability per level when degeneracies differ. For a ground level of degeneracy 1 and an excited level of degeneracy 2 at Δ=kBT ln2, the excited total weight is 2e^(−ln2)=1: the two levels each have probability 1/2. Classical Maxwell–Boltzmann assumptions differ from Bose–Einstein and Fermi–Dirac quantum occupations.

## Worked method

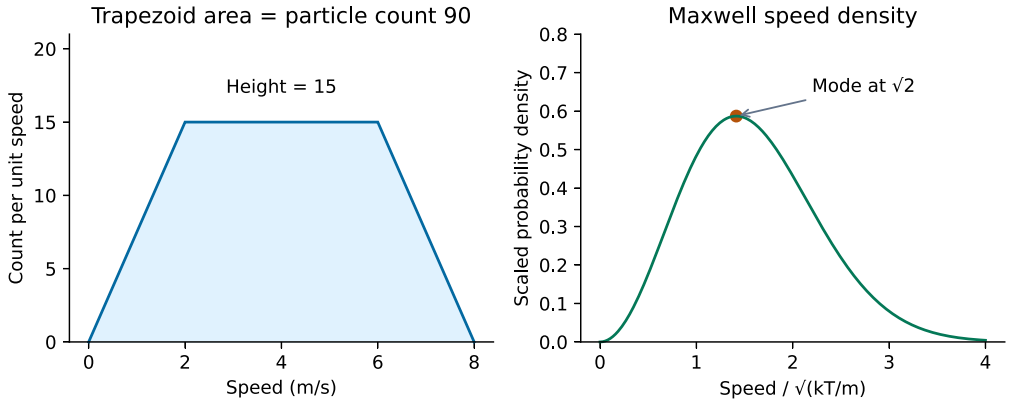
A canonical ensemble has ground degeneracy 1 and excited degeneracy g at gap Delta.

$$Z = 1 + g e^{-\Delta/(k_B T)}, \qquad P_e = \frac{g e^{-\Delta/(k_B T)}}{Z}.$$

With g = 4 and $\Delta = k_B T \ln 2$,

$$P_e = \frac{4 e^{-\ln 2}}{1 + 4 e^{-\ln 2}} = \frac{2}{3}.$$

This is probability for the entire excited level. Each of its four states has probability 1/6.




*The area under a distribution counts the particles; the Maxwell mode is √(2kT/m)*

## Check conditions and vocabulary

A density height is not a probability. Mean velocity, mean speed and mode differ; include degeneracy before normalising level probabilities.

**probability density**: Probability per unit of a continuous variable, whose integral gives interval probability.

**partition function**: Sum of statistical weights used to normalise equilibrium state probabilities.

# Counting statistics, uncertainty and dimensional models

GRE Subject Test · GRE Physics

A detector with ten percent efficiency does not register exactly ten photons out of every hundred. The count fluctuates even with stable efficiency.

Prerequisites: 47.

- Use binomial and Poisson count means and fluctuations
- Propagate small uncertainties with stated correlation assumptions
- Solve dimensional exponent constraints and check a model's units

## Choose the system and model

For N independent incident particles each detected with probability p, count X is binomial: mean Np and variance Np(1−p). The standard deviation is sqrt[Np(1−p)], not the variance itself. With large N and small p, a Poisson approximation 泊松近似 has mean $\lambda$=Np and variance $\lambda$, so deviation sqrt($\lambda$). For N=200 and p=0.05, the exact mean is 10 and deviation sqrt(9.5)≈3.08, while Poisson gives sqrt(10)≈3.16. Neither model promises a fixed count. Correlated detections or dead time can invalidate independence.

## Use the governing relation

For a smooth measured function y(x1,…), linearise changes using its partial derivatives. Independent small standard uncertainties combine in quadrature: $\sigma_y^2 \approx \Sigma(\partial y/\partial xi)^2 \sigma_i^2$. Correlated inputs require covariance cross terms. For y=x^a, fractional standard uncertainty 标准不确定度 is approximately |a|$\sigma$_x/|x|. Thus kinetic energy K=½mv$^2$ with negligible mass uncertainty has fractional uncertainty twice that of speed. With independent mass uncertainty, combine $(\sigma_m/m)^2+(2\sigma_v/v)^2$. These approximations need small errors and a suitable local linear model.

## Apply the conditions

Random scatter measures precision; a common calibration bias affects accuracy and does not disappear by averaging repeats. For independent repeated readings, the standard uncertainty of the mean falls as 1/sqrt(n), but shared systematic error does not. The uncertainty of a physical spread and uncertainty of its estimated mean are different quantities. State whether a quoted percentage is a standard uncertainty, confidence interval or worst-case bound. Summing absolute contributions is a conservative bound, not the independent-standard-error quadrature rule.

## Check the conclusion

Dimensional analysis equates powers of mass, length and time, rather than numerical sizes. For a Planck-length form G^a $\hbar$^b c^d, dimensions are [G]=L$^3$/(MT$^2$), [$\hbar$]=ML$^2$/T and [c]=L/T. Requiring length gives −a+b=0, 3a+2b+d=1 and −2a−b−d=0. Hence a=b=1/2 and d=−3/2, so length is sqrt(G$\hbar$/c$^3$). Dimensional analysis cannot determine an arbitrary dimensionless coefficient or prove that the chosen constants are physically sufficient. Reject a formula with wrong units before inserting numbers.

## Worked method

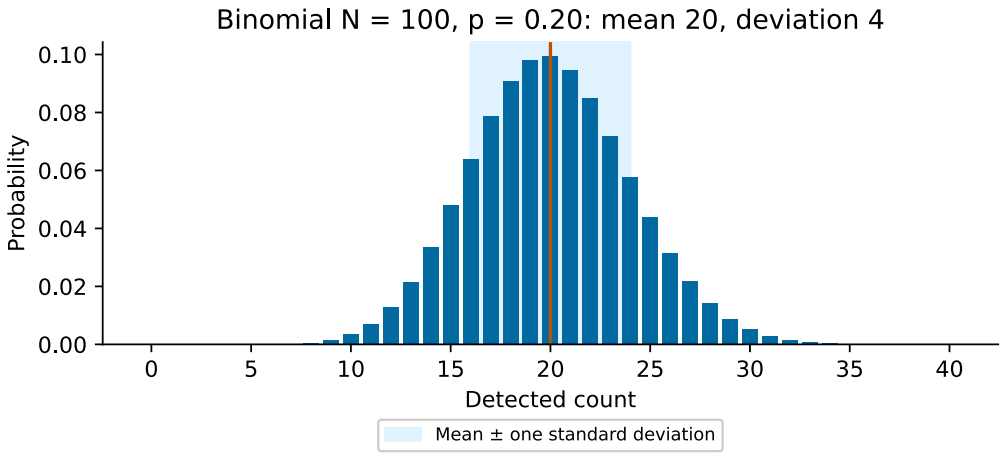
For independent small standard uncertainties in $K = mv^2/2$, differentiate first.

$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

With relative uncertainties 3% in mass and 2% in speed,

$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.




*Mean Np = 20, standard deviation √(Np(1 − p)) = 4*

## Check conditions and vocabulary

Do not confuse mean count with guaranteed count, variance with standard deviation, or a random standard error with a calibration bias. Dimensional consistency is necessary but not sufficient.

**standard uncertainty**: Uncertainty expressed as a standard deviation under a stated measurement model.

**Poisson approximation**: Rare independent-event count model with variance equal to its mean.



# Relativistic lifetime, energy and Doppler shift

GRE Subject Test · GRE Physics

A fast unstable particle can travel farther in the laboratory than its rest-frame lifetime multiplied by its speed would suggest.

Prerequisites: 6, 42.

- Relate proper lifetime to laboratory time and distance
- Use invariant energy–momentum and relativistic kinetic energy
- Infer longitudinal recession speed from wavelength ratio

## Choose the system and model

For relative speed v, define $\beta$=v/c and $\gamma$=1/sqrt(1−$\beta^2$). Proper time 固有时 is measured along the particle's worldline by a clock at rest with it. If its proper mean lifetime is $\tau$0, its laboratory mean lifetime is $\gamma\tau$0 and mean travel distance is v$\gamma\tau$0 at constant speed. The decay is statistical: mean lifetime is not a guaranteed decay time for each particle. At $\beta$=0.8, $\gamma$=5/3. Compute spacetime events consistently in one frame; proper time and coordinate time are different quantities.

## Use the governing relation

Total energy 总能量 is E=$\gamma$mc$^2$, momentum p=$\gamma$mv and invariant E$^2$−p$^2$c$^2$=m$^2$c$^4$. When E and pc are expressed in the same energy units, find mc$^2$=sqrt(E$^2$−(pc)$^2$), not E−pc. The nonnegative square root is required for positive rest mass. For E=13 GeV and pc=12 GeV, rest energy 静能 is 5 GeV and mass is 5 GeV/c$^2$. A massless particle has E=pc but need not have zero energy or momentum. These relations concern isolated-particle four-momentum, not classical mv at relativistic speed.

## Apply the conditions

Work accelerating a particle from rest equals kinetic energy K=E−mc$^2$=($\gamma$−1)mc$^2$. At $\beta$=0.6, $\gamma$=1.25 and K=0.25mc$^2$, while the classical ½mv$^2$ gives 0.18mc$^2$. Classical kinetic energy is the low-speed expansion and becomes inaccurate near c. Finite acceleration work increases $\gamma$ rather than allowing a massive particle to reach or exceed c. Keep total energy, rest energy and kinetic energy distinct when interpreting answer units.

## Check the conclusion

For purely longitudinal relative recession in special relativity, wavelength ratio r=λ_observed/λ_emitted=sqrt((1+β)/(1−β)). Rearranging gives β=(r²−1)/(r²+1). A ratio r=2 gives β=3/5, not c times r−1 from a low-speed approximation. Blueshift uses r<1 and a negative recession parameter under this convention. The formula assumes the shift is entirely kinematic; cosmological expansion or gravitational redshift requires a different model. State the question's stipulated model before interpreting a spectral ratio.

## Worked method

Total energy and momentum determine invariant rest energy.

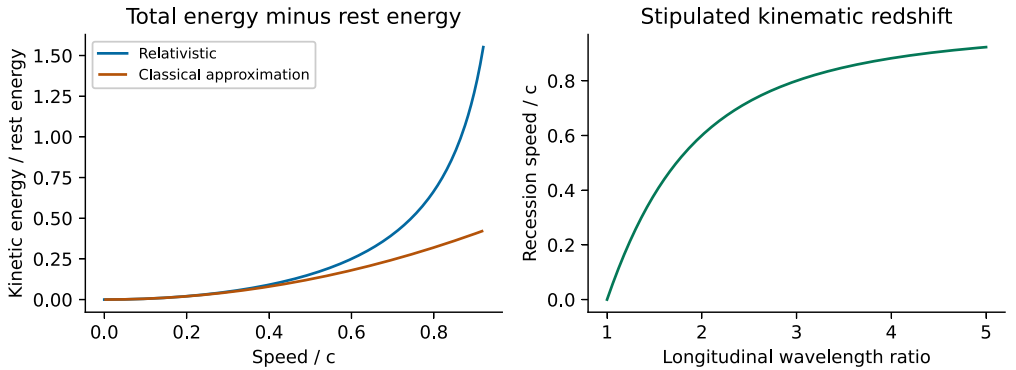
$$E^2 - (pc)^2 = (mc^2)^2, \qquad mc^2 = \sqrt{E^2 - (pc)^2}.$$

For E = 17 GeV and pc = 15 GeV,

$$mc^2 = \sqrt{(17\,\mathrm{GeV})^2 - (15\,\mathrm{GeV})^2} = 8\,\mathrm{GeV}.$$

$$K = E - mc^2 = 17\,\mathrm{GeV} - 8\,\mathrm{GeV} = 9\,\mathrm{GeV}.$$

Rest mass is 8 GeV/c squared. Do not subtract momentum directly from energy.




*Classical ½mv² underestimates kinetic energy at high speed; a wavelength ratio of 2 means 0.6c*

## Check conditions and vocabulary

Do not use the proper lifetime as laboratory time, subtract pc from E to find rest mass, or apply the classical Doppler approximation to a large wavelength ratio.

**proper time**: Time recorded by a clock moving with the object along its worldline.

**rest energy**: Energy mc² associated with an object's rest mass.



# Photoelectrons, reduced mass and atomic excitation

GRE Subject Test · GRE Physics

## Photoelectrons, reduced mass 约化质量 and atomic excitation

Repeated equally spaced current dips can represent repeated losses of the same excitation energy, rather than a sequence of different atomic levels.

Prerequisites: 7.

- Distinguish photoelectric thresholds and characteristic X rays
- Apply reduced-mass spectral scaling and many-electron spin filling
- Interpret Franck–Hertz energy spacing without inventing new levels

## Choose the system and model

Photoelectric maximum kinetic energy is Kmax=hν−φ for photon frequency above threshold ν0=φ/h. A stopping-potential magnitude satisfies eVs=Kmax, so Vs is linear in ν above threshold. Increasing intensity at fixed frequency increases the available photon count and usually photocurrent, not maximum photoelectron energy. Below threshold the simple one-photon model emits no photoelectrons regardless of intensity. Work function 逸出功 is a property of the surface, not proportional to the illumination frequency.

## Use the governing relation

Characteristic X rays result when an electron fills an inner-shell vacancy, emitting a photon equal to the shell energy difference. Their sharp lines depend on target atoms. Bremsstrahlung arises from deceleration of energetic electrons in nuclear electric fields and produces a continuous background, with an energy endpoint set by the incident electron energy. A continuous background and discrete lines can appear together; the existence of one does not exclude the other. These mechanisms differ from visible fluorescence, phonon scattering and particle capture.

## Apply the conditions

For a hydrogen-like two-body atom, replace electron mass by reduced mass μ=m_eM/(m_e+M). In the simple Coulomb model, level energies scale as −μZ²/n², so spectral frequencies scale as μZ² and wavelengths inversely. For positronium M=m_e, μ=m_e/2 and its Rydberg constant is half the infinite-nuclear-mass value. For a heavy nucleus μ approaches m_e. In many-electron atoms, use orbital filling and Hund's rule rather than the hydrogen model: degenerate orbitals are occupied singly with parallel spins before pairing. Carbon's 2p² gives two unpaired electrons

and total spin S=1; oxygen's $2p^4$ has two unpaired electrons and also S=1. Filled pairs contribute zero net spin.

## Check the conclusion

In Franck–Hertz experiments, accelerated electrons lose energy through inelastic excitation once they reach an atomic threshold. Repeated current-dip or peak spacing in accelerating voltage can therefore identify the same excitation energy lost one, two or more times. A spacing $\Delta$V corresponds to energy e$\Delta$V; peaks at 4,8,12 V need not represent three separate excited-level energies. Contact potentials and retarding fields can shift absolute peak positions, so use the spacing and stated apparatus conditions. If the excited atom returns by one photon of that energy, wavelength is hc/(e$\Delta$V).

## Worked method

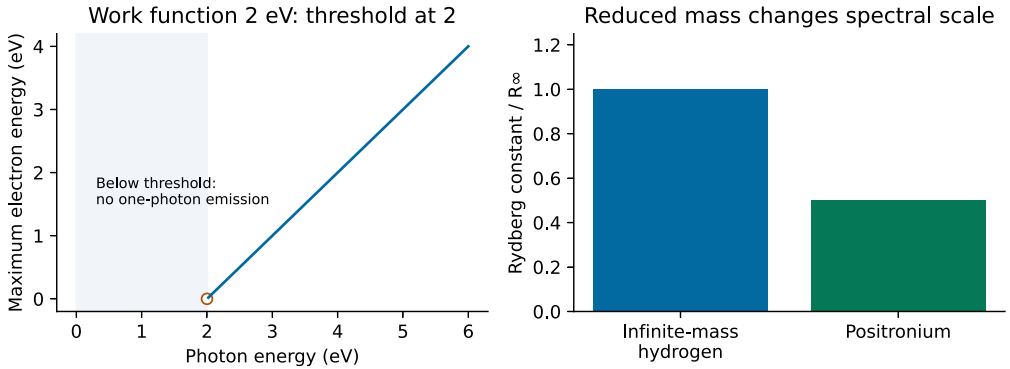
In the one-photon photoelectric effect 光电效应, use the surface work function phi.

$$K_{max} = h\nu - \phi, \qquad eV_s = K_{max}.$$

For photon energy 4.5 eV and phi = 2.0 eV,

$$K_{max} = E_\gamma - \phi = 4.5\,\text{eV} - 2.0\,\text{eV} = 2.5\,\text{eV}.$$

The stopping-potential magnitude is 2.5 V. Higher intensity at the same frequency changes the available electron rate, not this maximum energy.




*No emission below 2 eV; positronium's reduced mass halves its spectral scale*

## Check conditions and vocabulary

Current intensity and stopping voltage answer different questions. Include reduced mass and orbital degeneracy, and use Franck–Hertz spacing rather than treating every peak as a new energy level.

**work function**: Minimum energy required to remove an electron from the specified surface.

**reduced mass**: Two-body effective mass m1m2/(m1+m2) for relative motion.

# Scattering, degeneracy and Pauli operators

GRE Subject Test · GRE Physics

## Scattering, degeneracy 简并度 and Pauli operators

A quantum wave can reflect from a downward potential step even though a classical particle has no turning point there.

Prerequisites: 5.

- Match travelling waves and fluxes across finite potential changes
- Count degenerate isotropic-oscillator states
- Evaluate Pauli products and distinguish anticommutation from equality

## Choose the system and model

For constant potential V and energy E>V, the spatial solutions are travelling factors exp(±ikx), with k=sqrt[2m(E−V)]/ħ. At finite steps with the same particle mass, wavefunction and its first derivative are continuous. In a left-incident scattering problem with no incoming beam from the right, the far-right solution contains only the right-travelling factor Ae^(ikx). If a finite well returns to the original external potential, the transmitted external wave number equals the incident one, even though the interior wave number differs. Exponentially decaying solutions describe E<V regions, not every potential well.

## Use the governing relation

For a step from V1 to V2 with both regions classically allowed, write incident-plus-reflected amplitude in region 1 and transmitted amplitude in region 2. Continuity gives r=(k1−k2)/(k1+k2) and t=2k1/(k1+k2). Reflection probability is R=|r|²; transmission is T=(k2/k1)|t|² because probability current depends on wave number. Thus R+T=1 for a lossless step. Do not add raw squared transmitted amplitude to R without its current factor. With k2=3k1, R=1/4 and T=3/4 despite a downward step.

## Apply the conditions

A three-dimensional isotropic oscillator separates into x,y,z modes with nonnegative integers n_x,n_y,n_z. Its energy is (N+3/2)ħω, where N=n_x+n_y+n_z. For a spin-zero distinguishable single particle, the spatial degeneracy is the number of such triples: (N+1)(N+2)/2. This counts different assignments, not just different unordered partitions. For N=2, the six states are permutations of (2,0,0) and (1,1,0). For N=3, the degeneracy is 10. Additional spin or identical-particle constraints would change the counting problem.



## Check the conclusion

Pauli matrices are σ_x=[[0,1],[1,0]], σ_y=[[0,−i],[i,0]], σ_z=[[1,0],[0,−1]]. Each squares to identity. Direct multiplication gives σ_xσ_y=iσ_z and σ_yσ_x=−iσ_z, so distinct Pauli matrices anticommute. Cyclic products x→y→z have positive i; reversing order changes the sign. The general identity is σ_iσ_j=δ_ijI+iΣε_ijkσ_k. Matrix order matters: σ_xσ_z=−iσ_y, not iσ_y or simply σ_y. These dimensionless matrices become spin operators S_i=ħσ_i/2, adding physical units and factors.

## Worked method

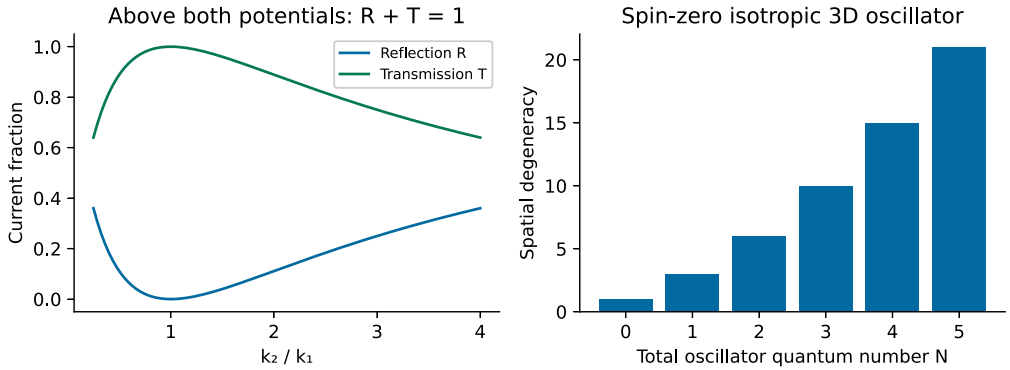
At a finite potential step, match wavefunction and derivative for the same particle mass.

$$r = \frac{k_1 - k_2}{k_1 + k_2}, \qquad t = \frac{2k_1}{k_1 + k_2}.$$

Transmission probability 透射概率 uses current, not amplitude alone. For $k_2 = 3k_1$, $r = -1/2$, $t = 1/2$.

$$R = |r|^2 = 1/4, \qquad T = (k_2/k_1)|t|^2 = 3/4, \qquad R + T = 1.$$

Reflection occurs even for a downward step. Its phase sign disappears when taking probability.




*$R + T = 1$, with $R = 0$ when $k_2 = k_1$; level $N$ has $(N + 1)(N + 2)/2$ states*

## Check conditions and vocabulary

Transmission probability needs the wave-number current ratio. Oscillator degeneracy counts ordered mode triples. Do not reverse Pauli matrix order without changing its sign.

**reflection coefficient** 反射系数: Reflected probability-current fraction relative to incident current.

**degeneracy**: Number of independent states sharing the specified energy.

# Nuclear binding, particle families and Hall carriers

GRE Subject Test · GRE Physics

Measuring conductivity alone does not tell you whether electrons or holes dominate transport. A magnetic Hall measurement provides a sign-sensitive test.

Prerequisites: 8, 46.

- Compare binding energy per nucleon 平均核子结合能 and reaction energy
- Distinguish leptons, mesons and baryons with conservation constraints
- Infer dominant carrier sign from the one-carrier Hall model

## Choose the system and model

Binding energy B is the energy needed to separate a nucleus into its constituent free nucleons, equal to the corresponding mass deficit times $c^2$. Binding energy per nucleon B/A is the relevant comparison across different mass numbers; a heavier nucleus can have greater total B but lower B/A. The broad B/A maximum is near the iron/nickel region. Light-nucleus fusion and very-heavy-nucleus fission can release energy when products have greater total binding. For a reaction, Q=(initial rest mass−final rest mass)$c^2$; positive Q means released energy. Always compare the actual specified isotopes and final products, not just their element names.

## Use the governing relation

Leptons such as electrons, muons and neutrinos are not made of constituent quarks in the Standard Model. A negative muon has charge −e and spin 1/2, like an electron, but has a different mass and belongs to a different flavour family; it is not a meson despite its historical name. Hadrons contain quarks: ordinary mesons have one quark and one antiquark, and ordinary baryons have three quarks. Corresponding antibaryons have three antiquarks. These elementary classification models do not imply every composite particle is a simple baryon or meson.

## Apply the conditions

Check charge, energy, momentum, baryon number and relevant lepton numbers in a proposed process. A proton's uud quarks sum to charge +e; neutron udd sums to zero. Beta-minus decay n→p+e−+antineutrino conserves electric charge and includes the antineutrino to balance lepton number and kinematics. A particle's mass alone does not classify it as a quark, lepton or hadron. Reaction thresholds may require kinetic energy beyond an exothermic rest-mass difference when other constraints apply.



## Check the conclusion

In a simple one-carrier conductor, transverse magnetic Lorentz force builds a Hall electric field until transverse carrier force balances. With a stated tensor/sign convention, Hall coefficient 霍尔系数 is R_H=1/(nq); negative q gives electron-like negative coefficient, positive q gives hole-like positive coefficient. Resistivity magnitude alone does not reveal this sign. Set current, magnetic field and voltage directions consistently before reading raw polarity. Semiconductors with electrons and holes both contributing require a mobility-weighted two-carrier model; the simple coefficient need not equal the inverse total carrier density.

## Worked method

In a one-carrier Hall effect 霍尔效应 model, use $R_H = 1/(nq)$ with a fixed lead convention. For electrons, q = -e, n = $5.0 \times 10^{22}$ per cubic metre,

$$R_H = -\frac{1}{ne} = -\frac{1}{(5.0 \times 10^{22}\,\mathrm{m^{-3}})(1.60 \times 10^{-19}\,\mathrm{C})} = -1.25 \times 10^{-4}\,\mathrm{m^3/C}.$$

Negative coefficient indicates electron-like carriers under this model, not a negative density. Raw voltage polarity is meaningful only with declared field and lead directions.

Particle families in the ordinary constituent model

| Examples | Ordinary constituent model | Family |
|---|---|---|
| Electron / muon | None | Lepton |
| Pion / kaon | Quark + antiquark | Meson |
| Proton / neutron | Three quarks | Baryon |

*Classify a particle by its constituents: leptons, mesons or baryons*

## Check conditions and vocabulary

Compare binding per nucleon, not just total binding. A muon is a lepton, and Hall sign requires the declared measurement convention and carrier model.

**binding energy per nucleon**: Nuclear binding energy divided by nucleon count.

**Hall coefficient**: Signed proportionality relating transverse Hall field to current density and magnetic field under a stated convention.

# Constraints, variational equations and cyclic coordinates

GRE Subject Test · GRE Physics

A pendulum tension changes throughout a swing, but its angular equation can be derived without solving for tension.

Prerequisites: 1, 47.

- Derive Euler–Lagrange motion after imposing a holonomic constraint 完整约束
- Use a cyclic coordinate 循环坐标 to identify conserved canonical momentum
- Linearise a stable equilibrium and check the resulting frequency

## Choose independent coordinates

A constraint removes an independent coordinate before you form the kinetic energy. For a fixed-length pendulum choose $\theta$ from the downward vertical: x=l sin$\theta$ and y=−l cos$\theta$. Differentiating both coordinates gives $v^2=l^2\theta dot^2$, not $l^2$ $\sin^2\theta$ $\theta dot^2$. Thus $T=ml^2\theta dot^2/2$ and V=mgl(1−cos$\theta$), with zero potential at the bottom. The fixed length is a holonomic constraint: it is a relation among coordinates and possibly time. A rolling velocity constraint needs its own analysis; do not automatically treat every constraint as a coordinate substitution.

## Derive the equation

For ideal constraints and the usual conservative system, the stationary-action equation is d/dt($\partial$L/$\partial$qdot)−$\partial$L/$\partial$q=0 with L=T−V. The variations vanish at the two time endpoints. A pendulum gives $\partial L/\partial\theta dot=ml^2\theta dot$ and $\partial L/\partial\theta=-mgl\sin\theta$, so $ml^2\theta ddot+mgl\sin\theta=0$. The momentum derivative acts on every time-dependent factor: for a varying radius, $d(mr^2\theta dot)/dt$ contains 2mr rdot $\theta$dot. The variational method does not mean mechanical energy is conserved in a time-dependent system.

## Find cyclic momentum

For planar central motion, $L=m(rdot^2+r^2\theta dot^2)/2-V(r)$. The angle $\theta$ is cyclic because L has no explicit $\theta$ dependence, even though L depends on $\theta$dot. Its canonical momentum $p\theta=mr^2\theta dot$ is constant. The radial equation is m rddot=mr$\theta dot^2-dV/dr$. The first term is part of the coordinate acceleration, not a new outward real force in an inertial frame. Conservation of p$\theta$ makes angular speed increase as r decreases; constant angular speed is a different, externally driven situation.

## Linearise with conditions

Near a stable equilibrium q0, expand V to second order and use a constant local inertia M: $V\approx V(q0)+V''(q0)(q-q0)^2/2$. Then the small-displacement frequency is sqrt(V″/M). For a pendulum $\sin\theta\approx\theta$ in radians, so $\omega$=sqrt(g/l); the approximation requires small amplitude. A negative V″ gives instability, not an oscillation with an imaginary measurable frequency. Check the full nonlinear equation first, then state the approximation and compare units: V″/M must have units of inverse time squared.

## Worked method

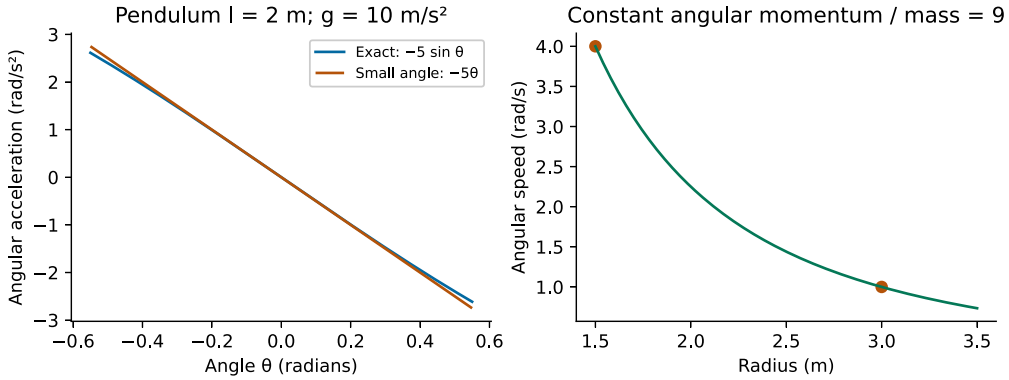
For a fixed pendulum length l, use theta from downward vertical: x = l sin theta, y = -l cos theta. Both Cartesian velocities give $T = ml^2\dot{\theta}^2/2$ and $V = mgl(1-\cos\theta)$. The Euler-Lagrange equation 欧拉 – 拉格朗日方程 is

$$\frac{d}{dt}(ml^2\dot{\theta}) + mgl\sin\theta = 0.$$

Divide by $ml^2$ before linearising:

$$\ddot{\theta} + (g/l)\sin\theta = 0, \qquad \omega = \sqrt{g/l} \quad (|\theta| \ll 1).$$

The approximation uses radians. Endpoint variations vanish in the stationary-action derivation.




*Small angles make the pendulum linear ($\ddot{\theta} \approx -5\theta$); constant angular momentum makes $\omega$ fall as $1/r^2$*

## Check conditions and vocabulary

A cyclic coordinate is absent from L itself; its velocity need not be absent. Do not discard the time derivative of a changing $r^2$ factor.

**holonomic constraint**: A constraint written as a relation among coordinates and time.

**cyclic coordinate**: Absent from L itself, so its canonical momentum is conserved.

# Accelerating and rotating reference frames

GRE Subject Test · GRE Physics

A person standing still on a turning platform needs a real inward force, although their platform-relative acceleration is zero.

Prerequisites: 26.

- Transform acceleration with translating and rotating frame 旋转参考系 terms
- Determine centrifugal, Coriolis and Euler directions from cross products
- Separate apparent forces from real interactions and test inertial limits

## Translate the frame

In a translating frame with origin acceleration A, Newton's equation becomes m a′=F_real−mA. An upward-accelerating elevator therefore has N−mg=ma in the ground frame, or N−mg−ma=0 for its stationary passenger in the elevator frame. These are the same prediction. The additional term is an apparent force caused by the chosen accelerating coordinates; it is not a new contact with another body. Uniform translation with A=0 changes velocity but needs no apparent force.

## Differentiate rotating axes

For a rotating basis, differentiating a vector adds Ω×that vector. Applying this twice gives a=a_origin+a′+2Ω×v′+Ω×(Ω×r)+Ωdot×r. Here r and v′ are measured relative to the moving origin in its rotating axes; all vectors in a calculation must be expressed in the same basis at the same instant. Move the last three rotational terms to the force side to obtain Coriolis −2mΩ×v′, centrifugal −mΩ×(Ω×r), and Euler −mΩdot×r. A constant rotation removes the Euler term, not the Coriolis term.

## Check cross-product directions

Take Ω along +z, an anticlockwise platform viewed from above, and a particle at r along +x. Centrifugal force 离心力 is along +x with magnitude mΩ²r. If the particle moves outward with v′ along +x, then Ω×v′ is +y, so Coriolis force 科里奥利力 points −y. Reversing the relative velocity reverses Coriolis; keeping v′=0 makes it vanish. Coriolis is perpendicular to v′ and does no instantaneous work on that relative motion. Centrifugal force is outward from the rotation axis, not necessarily outward from the chosen origin in an arbitrary three-dimensional position.

## Balance real and apparent forces

A body at rest on the platform has a′=v′=0. With constant rotation and a fixed origin, its real force must be mΩ×(Ω×r), inward, cancelling the outward apparent term in the rotating equation. If rotation changes, a tangential real force must also balance the Euler term. Always check Ω→0 and A→0: ordinary inertial Newtonian motion must return. Do not mix the real inward centripetal requirement with an added outward real reaction on the same body; interaction partners belong in separate free-body diagrams.

## Worked method

In a rotating frame, Coriolis force uses the relative velocity.

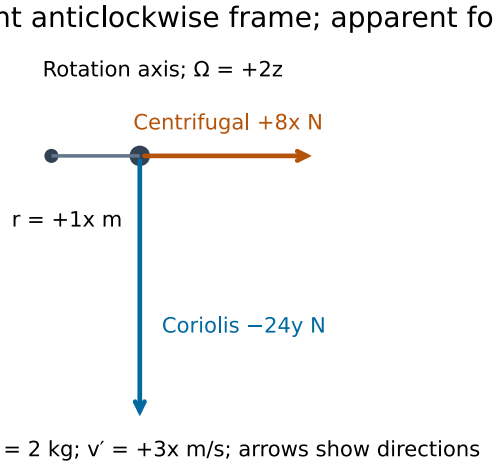
$$\mathbf{F}_C = -2m\boldsymbol{\Omega} \times \mathbf{v}', \qquad \mathbf{F}_{cf} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

For m = 2 kg, Omega = 2 rad/s along z, r = 1 m along x and v' = 3 m/s along x,

$$F_{cf} = m\Omega^2 r = (2\,\mathrm{kg})(2\,\mathrm{rad/s})^2(1\,\mathrm{m}) = 8\,\mathrm{N} \quad (+x).$$

$$F_C = 2m\Omega v' = 2(2\,\mathrm{kg})(2\,\mathrm{rad/s})(3\,\mathrm{m/s}) = 24\,\mathrm{N} \quad (-y).$$

Euler force is zero only because Omega is constant. These are coordinate terms, not extra physical contacts.




*Centrifugal force $m\Omega^2 r$ = 8 N; Coriolis force $-2m\Omega \times v'$ = −24 N on y*

## Check conditions and vocabulary

The relative velocity v′ belongs in the Coriolis term. Using the full inertial velocity counts rotation twice.

**Coriolis force**: The apparent rotating-frame force −2mΩ×v′ caused by relative motion.

**centrifugal force**: The apparent rotating-frame force −mΩ×(Ω×r), directed away from the rotation axis.

# Fluid continuity, pressure energy and viscous flow

GRE Subject Test · GRE Physics

## Fluid continuity 连续性, pressure energy and viscous flow

A narrow pipe carries the same steady flow rate as a wide pipe, but its pressure behaviour depends on both speed and viscous losses.

Prerequisites: 10, 47.

- Combine incompressible continuity with pressure, height and kinetic energy
- Apply hydrostatic and Bernoulli relations only under their stated conditions
- Use viscosity, Poiseuille scaling and Reynolds number to distinguish flow regimes

## Conserve the flow rate

For steady flow, the mass passing successive cross-sections per unit time is equal: $\rho$Av is constant. An incompressible liquid has nearly constant $\rho$, so Q=Av is the volume flow rate 体积流量. Halving pipe radius quarters area and multiplies mean speed by four for the same Q. Pressure does not determine Q without a model of the rest of the system. In a stationary liquid, dp/dz=$-\rho$g with z upward; therefore pressure increases by $\rho$gh a distance h below a surface. Use absolute or gauge pressure consistently on both sides.

## Account for pressure energy

For steady, incompressible, inviscid motion along a streamline with no pump or dissipative loss, Bernoulli gives p+$\rho v^2/2+\rho$gz=constant. These three terms are energy per volume and have pressure units. At equal height, larger speed requires smaller static pressure under these assumptions. A higher outlet also uses pressure/kinetic energy to gain gravitational energy. Bernoulli along one streamline need not imply the same constant on different streamlines in rotational flow. A stagnation point has v=0 and converts local speed energy to a pressure rise in the ideal model.

## Include viscous loss

Viscosity transports momentum between neighbouring fluid layers. For steady fully developed laminar flow of a Newtonian incompressible liquid in a circular tube, Q=$\pi R^4\Delta p/(8\eta L)$. Here $\Delta$p is the pressure drop along the tube, $\eta$ dynamic viscosity 动力黏度 and L tube length. Doubling radius at fixed $\Delta$p, $\eta$ and L multiplies Q by sixteen. Doubling Q at fixed geometry needs twice the pressure drop. This viscous pressure loss cannot be added to a loss-free Bernoulli equation as though nothing changes; include a dissipative pressure/head loss or use the viscous model.



## Check the flow regime

The Reynolds number Re=$\rho$vD/$\eta$ compares inertial and viscous effects using characteristic speed v and length D. For a pipe use mean speed and internal diameter, not radius. Re is dimensionless: density times speed times length has the same units as dynamic viscosity. Small Re favours viscous dominance; transition to turbulence depends on geometry and disturbances, so a single threshold is not a universal law. Poiseuille scaling is not safe after assuming a turbulent flow. Check volume continuity, sign of pressure change and the regime before selecting a formula.

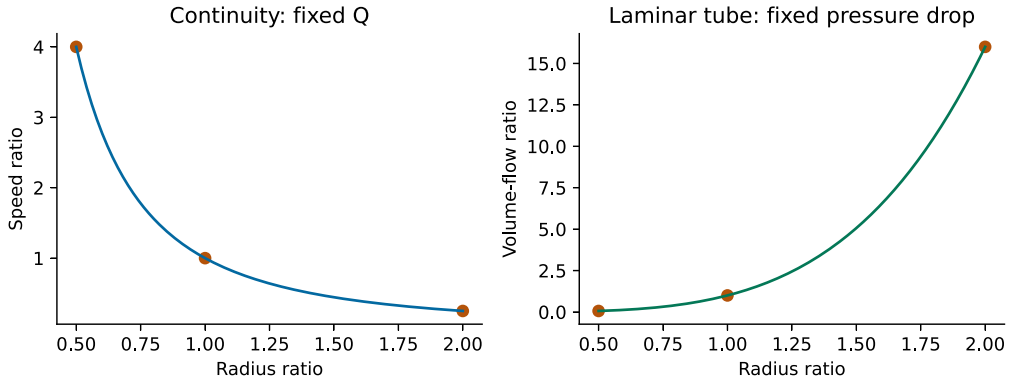
## Worked method

A steady incompressible horizontal ideal flow has areas 4 and 1 square centimetres. At the first section speed is 1 m/s. Continuity and Bernoulli give

$$v_2 = A_1 v_1 / A_2 = (4\,\mathrm{cm}^2)(1\,\mathrm{m/s})/(1\,\mathrm{cm}^2) = 4\,\mathrm{m/s}.$$

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) = \tfrac{1}{2}(1000\,\mathrm{kg/m}^3)[(4\,\mathrm{m/s})^2 - (1\,\mathrm{m/s})^2] = 7500\,\mathrm{Pa}.$$

These relations assume no pump, viscosity loss or height change. A real narrow tube may violate them.




*Continuity gives $v \propto 1/r^2$; Poiseuille flow gives $Q \propto r^4$*

## Check conditions and vocabulary

Continuity keeps Q constant across one steady pipe; Poiseuille compares Q between different systems at a specified pressure drop. State what is held fixed before comparing radius powers.

**volume flow rate**: Volume crossing a section per unit time, Q=Av for mean speed v.

**dynamic viscosity**: The coefficient relating shear stress to velocity gradient in a Newtonian fluid.

# Polarisation, magnetisation and material fields

GRE Subject Test · GRE Physics

## Polarisation, magnetisation 磁化强度 and material fields

Putting a dielectric into a capacitor changes the field differently depending on whether its battery remains connected.

Prerequisites: 13, 15.

- Relate free and bound charge to D, E and polarisation
- Compare fixed-charge and fixed-voltage dielectric changes
- Use magnetic constitutive response and free-current boundary conditions

## Separate free and bound charge

Electric polarisation 电极化强度 P is electric dipole moment per unit volume, measured in C/m$^2$. Define D=$\varepsilon_0$E+P so that $\nabla \cdot$ D=$\rho$_free; the total charge, including bound charge, still appears in $\nabla \cdot$ E=$\rho$_total/$\varepsilon_0$. Bound volume charge is $\rho$_b=$-\nabla \cdot$ P and bound surface charge is $\sigma$_b=P $\cdot$ n, where n points outward from the material. Uniform P therefore gives no bound charge in the bulk but can give surface charges of opposite sign. A dielectric is not an ideal metal: polarisation need not make its internal electric field zero.

## Apply the material relation

For a linear isotropic dielectric, P=$\varepsilon_0\chi$_e E and D=$\varepsilon$E with $\varepsilon$=$\varepsilon_0$(1+$\chi$_e)=$\varepsilon_0\varepsilon$_r. These simple scalar relations assume the response is linear and ignore anisotropy, strong dispersion and nonlinear effects. In a fully filled, large parallel-plate capacitor with negligible edge effects, D normal to the plates equals the free surface-charge density. If $\sigma$_free stays fixed, increasing $\varepsilon$ reduces E=$\sigma$_free/$\varepsilon$. The bound charges oppose the applied field; their magnitude is not automatically equal to the free plate charge.

## Hold charge or voltage fixed

For plate area A and spacing d, C=$\varepsilon$A/d. Disconnecting the battery fixes free charge Q: V=Q/C falls and stored energy U=Q$^2$/(2C) falls when a dielectric increases C. Leaving an ideal voltage source connected fixes V: Q=CV and stored energy U=CV$^2$/2 both rise. The source supplies energy and moving the dielectric can involve mechanical work; compare the stated electrical energy quantity rather than assuming the capacitor alone is an isolated system. At a material interface n $\cdot$ (D$_2$$-$D$_1$)=$\sigma$_free, whereas static tangential E is continuous. Normal E generally changes when permittivity changes.



## Distinguish magnetic fields

Magnetisation M is magnetic dipole moment per unit volume, measured in A/m. In SI, B=$\mu_0$(H+M); for a linear isotropic response M=$\chi$_m H and B=$\mu_0$(1+$\chi$_m)H. B and H have different units and roles. A long uniform solenoid has H≈nI when end and demagnetising effects are negligible; material response then changes B. Free surface current sets n$\times$(H$_2$$-$H$_1$)=K_free, and normal B remains continuous. Ferromagnetic hysteresis and saturation cannot be represented by one constant $\chi$_m over every field.

## Worked method

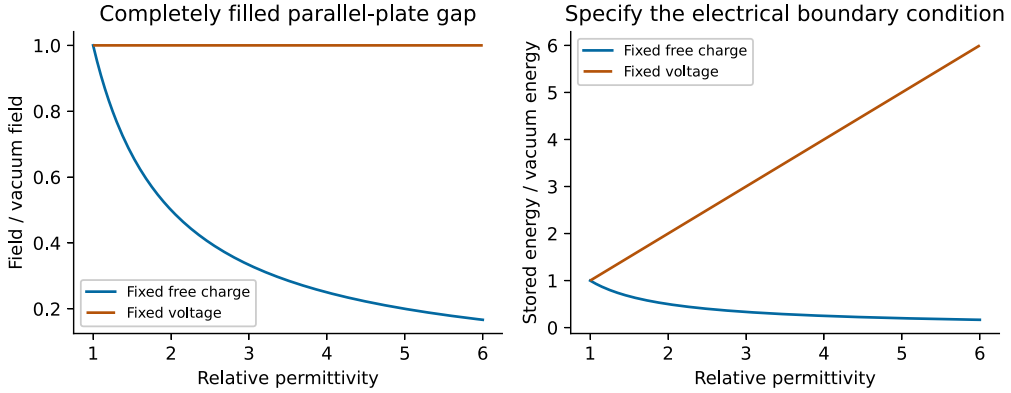
A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance 电容 becomes four times its original value.

$$V' = Q/(4C) = V/4, \qquad U' = Q^2/(2\times 4C) = U/4.$$

If an ideal battery stays connected instead, V is fixed:

$$Q' = 4CV = 4Q, \qquad U' = \tfrac{1}{2}(4C)V^2 = 4U.$$

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.




*At fixed charge a dielectric lowers E and energy by $\kappa$; at fixed voltage the energy rises by $\kappa$*

## Check conditions and vocabulary

Use free charge in the D equation and total charge in the E equation. Do not confuse fixed Q with fixed V, or identify B numerically with H.

**electric polarisation**: Electric dipole moment per unit volume, P, contributing bound charge.

**magnetisation**: Magnetic dipole moment per unit volume, M, entering B=$\mu_0$(H+M).

# RC and RL transients, impedance and resonance

GRE Subject Test · GRE Physics

## RC and RL transients, impedance 阻抗 and resonance

A capacitor voltage and an inductor current cannot change instantly under finite ordinary circuit drives, but their steady-state behaviour is different.

Prerequisites: 14.

- Solve RC charging and discharging with stated initial conditions
- Solve RL current response and account for stored energy
- Use complex impedance, phase and resonance in sinusoidal circuits

## Solve the capacitor response

For a resistor R in series with a capacitor C and a DC source V_s, Kirchhoff's voltage law is R dq/dt+q/C=V_s. Define V_C=q/C and $\tau$=RC. After a switch to a constant source, V_C(t)=V_s+(V_C(0)−V_s)exp(−t/$\tau$). An initially uncharged capacitor therefore has V_C=V_s(1−exp(−t/$\tau$)) and charging current I=(V_s/R)exp(−t/$\tau$). With the source removed and the resistor connected across it, V_C=V_C(0)exp(−t/$\tau$). Capacitor voltage is continuous across the switch if there is no impulsive current; the resistor current may change instantly.

## Track inductor current and energy

In a series RL circuit, Kirchhoff's law gives L dI/dt+RI=V_s and $\tau$=L/R. The current after a constant drive is I(t)=V_s/R+(I(0)−V_s/R)exp(−t/$\tau$). Inductor current is continuous for a finite voltage; its voltage may change abruptly as switching changes dI/dt. At late times an ideal inductor in a DC circuit acts as a zero-voltage connection, while an ideal capacitor has zero DC current. Stored energies are $LI^2/2$ and $CV_C^2/2$. During decay into a resistor, the initially stored energy becomes resistor heat rather than disappearing when the source is disconnected.

## Use the phasor convention

Use the declared phasor convention exp(i$\omega$t). A resistor has impedance R, an inductor i$\omega$L and a capacitor 1/(i$\omega$C)=−i/($\omega$C). A series RLC circuit therefore has Z=R+iX with X=$\omega$L−1/($\omega$C). Divide the source voltage phasor by Z to obtain current. Its magnitude is V_rms/sqrt($R^2$+$X^2$) when rms quantities are used. The impedance phase $\varphi$ satisfies tan$\varphi$=X/R; current lags voltage for X>0 and leads it for X<0. Adding the scalar magnitudes R, $\omega$L and 1/($\omega$C) loses the vector phase information.



## Separate resonance from decay

Series resonance occurs at $\omega_0$=1/sqrt(LC) when X=0. Current is maximal for a fixed voltage in this ideal series model, and the voltage/current phase difference is zero. Mean real power is V_rms I_rms cos$\varphi$=$I_rms^2$R; ideal L and C exchange energy but dissipate no average power. This differs from transient natural frequency: a damped series circuit can oscillate at sqrt(1/(LC)−(R/(2L))$^2$) when underdamped. Large resistance removes such free oscillation but does not change the condition X=0 of the ideal driven series impedance. State whether a question asks for a step response, free decay or sinusoidal steady state.

## Worked method

An RC circuit switches to fixed source voltage Vs. Charge conservation and Kirchhoff's law give

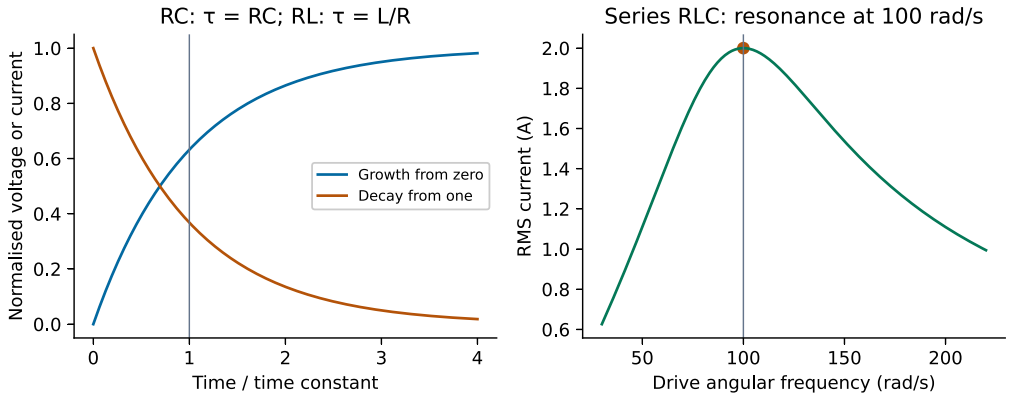
$$RC\dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s]e^{-t/(RC)}.$$

For R = 2 kilohms, C = 1 millifarad, Vs = 12 V and initial voltage zero,

$$\tau = RC = (2000\,\Omega)(0.001\,\mathrm{F}) = 2.0\,\mathrm{s}.$$

$$V_C(\tau) = V_s(1 - e^{-1}) = (12\,\mathrm{V})(1 - e^{-1}) = 7.59\,\mathrm{V}.$$

The time constant 时间常数 controls the remaining difference from the final state.




*$\tau$ = RC or L/R; a series RLC circuit draws its largest current at resonance*

## Check conditions and vocabulary

RC has $\tau$=RC, while RL has $\tau$=L/R. Do not confuse current amplitude with rms current, or resonance of a driven impedance with damped free-oscillation frequency.

**time constant**: The exponential response scale, RC for an RC circuit and L/R for an RL circuit.

**impedance**: The complex voltage-to-current phasor ratio in sinusoidal steady state.

# Polarisation, analyser chains and phase

GRE Subject Test · GRE Physics

Inserting a third polariser 偏振片 between crossed filters can increase the transmitted intensity, even though every filter removes energy.

Prerequisites: 3.

- Calculate successive ideal analyser transmissions from the immediate input state
- Distinguish linear, circular and elliptical field motion using relative phase
- Use Brewster incidence with refractive-index and plane-of-incidence conventions

## Project the electric field

For propagation along z, a transverse electric field can have x and y components. Linear polarisation means the field oscillates along one fixed line; an ideal analyser transmits the projection along its axis. Field amplitude becomes E cos$\theta$, so intensity becomes I cos$^2\theta$. Here $\theta$ is between the incoming polarisation and that analyser, and I is the intensity immediately before it. An unpolarised beam is a statistical mixture of transverse orientations; an ideal first polariser passes half its mean intensity. The outgoing beam is then linearly polarised along the filter axis. Do not apply a new factor of one half at every later filter, or use one angle to the original source for the whole chain.

## Update each analyser input

After each analyser, update both the intensity and the polarisation direction. For an initially unpolarised beam and ideal axes $\alpha_1$, $\alpha_2$, $\alpha_3$, the final intensity is $(I_0/2)\cos^2(\alpha_2-\alpha_1)\cos^2(\alpha_3-\alpha_2)$. Two crossed filters transmit zero in this model; a middle oblique axis changes the direction before the final projection. This increase relative to the crossed pair does not create energy: every step still has transmission between zero and one. Real filters have absorption, imperfect extinction and wavelength dependence. Use the ideal law only when those losses are excluded or separately specified.

## Track transverse phase

Write E_x=A cos$\omega$t and E_y=B cos($\omega$t+$\delta$) at a fixed point. Equal or opposite phases give a line, including a line at an oblique angle; equal nonzero amplitudes and a quarter-cycle phase difference give a circle. Unequal amplitudes at quarter-cycle phase give an ellipse. A quarter-wave plate adds a relative phase of $\pi/2$ between its principal axes under its design conditions. A linear input at 45° supplies equal components, so the outgoing field can be circular. A linear input along a principal axis has only one component and remains linear. For circular input, every ideal linear analyser passes half the intensity; handedness requires a declared viewing direction and phase convention.

## Apply Brewster geometry

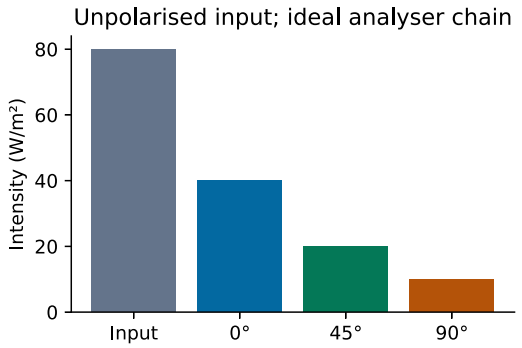
For incidence from transparent nonmagnetic medium $n_1$ into $n_2$, Brewster's angle measured from the normal satisfies tan$\theta$_B=$n_2/n_1$. At that angle, the reflected p component, whose electric field lies in the plane of incidence, vanishes in the ideal dielectric model. Reflected light from unpolarised input is then s polarised, perpendicular to that plane. Snell's law gives a refracted angle complementary to $\theta$_B. This is distinct from the critical-angle condition sin$\theta$_c=$n_2/n_1$, which needs $n_1>n_2$ and concerns total internal reflection. Brewster reflection does not imply that the entire incident intensity is reflected or that the transmitted beam is completely polarised.

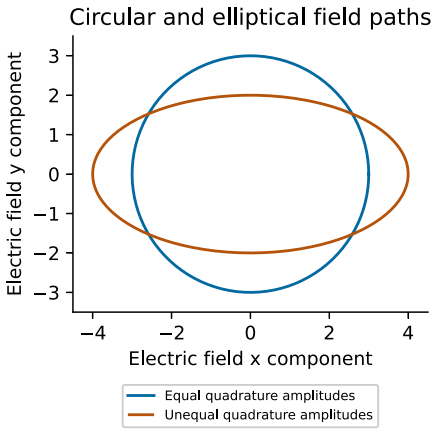
## Worked method

An ideal first polariser transmits half of unpolarised intensity I0. Each later analyser uses the angle from the preceding axis. For axes at 0°, 30° and 90°,

$$I_f = \tfrac{1}{2} I_0 \cos^2(30^\circ)\cos^2(60^\circ) = 3I_0/32.$$

Each filter re-aligns the field to its own axis; using only the first and last axes wrongly gives zero.




*The first polariser halves unpolarised light; each later one applies Malus's law, cos$^2\theta$*

## Check conditions and vocabulary

A phase plate changes relative phase; a polariser removes a field component.

**Malus's law** 马吕斯定律: I_out = I_in cos$^2\theta$ for an ideal linear analyser.

**Brewster angle** 布儒斯特角: Incidence angle where the reflected p component vanishes.

# Diffraction envelopes and missing interference orders

GRE Subject Test · GRE Physics

## Diffraction envelopes and missing interference 干涉 orders

Two equal slits can have the right path difference for constructive interference and still deliver no intensity at that angle.

Prerequisites: 3, 17.

- Derive and evaluate the far-field single-slit intensity using amplitude superposition
- Separate double-slit interference spacing from finite-aperture envelope 包络线 width
- Identify missing orders and state the limits of far-field and small-angle formulas

## Add aperture amplitudes

For a uniformly illuminated slit of width a, far-field contributions across its opening arrive with a phase gradient k sin$\theta$. Add their complex amplitudes before squaring: the normalised integral over x from $-$a/2 to a/2 is sin$\beta/\beta$, where $\beta$=$\pi$a sin$\theta/\lambda$. The intensity ratio is $(\sin\beta/\beta)^2$. At $\theta$=0 take the limit sin$\beta/\beta\to$1, rather than calling the centre undefined or dark. Zeros occur at a sin$\theta$=m$\lambda$ with nonzero integer m. The central peak lies between the first zeros and is twice as wide as one adjacent zero-to-zero interval in sin$\theta$. Side-peak maxima are not exactly halfway between their zeros.

## Convert angles to the screen

At small angles on a distant screen L away, y$\approx$L$\theta$ and sin$\theta\approx\theta$ give first zeros y$\approx\pm$L$\lambda$/a. Thus the central width is 2L$\lambda$/a. Use metres consistently: a millimetre slit and a nanometre wavelength differ by six powers of ten. For a wider angle, use $\theta$=asin(m$\lambda$/a) and y=L tan$\theta$; the small-angle y expression then becomes inaccurate. Far-field conditions need nearly parallel rays from different parts of the aperture, for example L much greater than $a^2/\lambda$, or the equivalent focal-plane arrangement. Near-field Fresnel patterns cannot be assigned this intensity law blindly.

## Multiply interference and envelope

For two coherent identical uniformly illuminated slits of width a and centre separation d, the normalised pattern is $(\sin\beta/\beta)^2\cos^2\alpha$, with $\alpha$=$\pi$d sin$\theta/\lambda$ and central intensity as the normalisation. The $\cos^2$ factor produces interference orders d sin$\theta$=n$\lambda$; the single-slit factor gives the broader envelope zeros. Small-angle neighbouring interference spacing is $\lambda$L/d. Increasing d narrows fringe spacing; increasing a narrows the envelope. These are separate changes. The equal-height narrow-slit interference formula alone does not predict the diminishing brightness or missing orders of finite apertures. Incoherent sources do not maintain the same phase-dependent cross term.

## Cancel coincident orders

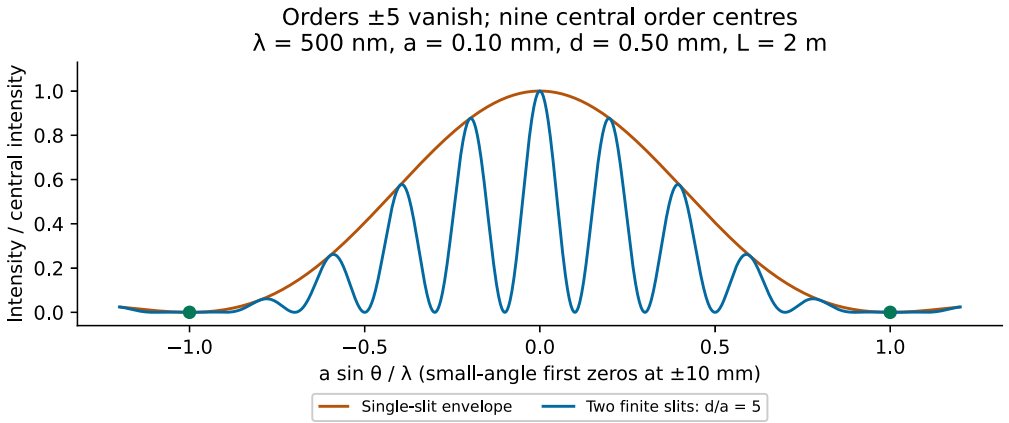
A missing order 缺级 occurs when n$\lambda$/d=m$\lambda$/a simultaneously, giving n=m d/a. If d/a is an integer r, orders $\pm$r, $\pm$2r and so on are cancelled by aperture zeros. Do not count an order at an envelope boundary as a visible bright fringe. In the central envelope, the nominal interference-order centres satisfy |n|<d/a; when r is an integer there are 2r$-$1 such centres. For finite slit width, exact local maxima are shifted slightly by the changing envelope, so the interference-order locations are an approximation to observed peak centres. Identify what quantity is being requested before treating every $\cos^2$ maximum as an exact maximum of the product.

## Worked method

For two finite slits, separate interference spacing from the single-slit envelope.

$$I/I_{centre} = (\sin\beta/\beta)^2\cos^2\alpha, \quad \beta = \pi a\sin\theta/\lambda, \quad \alpha = \pi d\sin\theta/\lambda.$$

If d = 5a, the first envelope zero $a\sin\theta = \lambda$ coincides with order n = 5. Nominal order centres in the central envelope are n = -4 through +4: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.




*With d/a = 5 the fifth orders fall on envelope zeros, leaving nine central fringes*

## Check conditions and vocabulary

Add field amplitudes before squaring, distinguish a from d, and exclude dark boundary orders. An interference maximum alone does not guarantee a maximum of the full pattern.

**diffraction envelope** 衍射包络: The aperture-dependent intensity factor that modulates the interference pattern.

**missing order**: An interference order cancelled because it coincides with an aperture intensity zero.



# Signed lens and mirror images

GRE Subject Test · GRE Physics

A negative image distance is a geometric result that locates a virtual image 虚像, not a failed calculation to replace by its absolute value.

Prerequisites: 3.

- Locate paraxial thin-lens and spherical-mirror images with a declared sign convention
- Use signed magnification to classify orientation and real or virtual character
- Track successive image locations as the next element's object without resetting geometry

## Declare distances and signs

Use the real-is-positive convention for this unit: a real object sends diverging incident rays into the element, so $s>0$; an already converging incident beam can represent a virtual object, $s<0$. A real image has $s'>0$, while a virtual image has $s'<0$. The paraxial equation is $1/f=1/s+1/s'$ and transverse magnification 横向放大率 $M=-s'/s$. A converging lens 会聚透镜 has $f>0$ and a diverging lens $f<0$. For left-to-right light and a real object at $x<0$ relative to a lens at $x=0$, a real image lies at $x=s'>0$; a negative $s'$ puts its virtual image on the input side. State the convention before substitution, as other signed-coordinate conventions give different-looking equations.

## Trace a lens image

For a converging thin lens, a ray parallel to the axis exits through the far focal point, and a ray through the ideal optical centre continues undeviated. Their intersection identifies a real image; backwards extensions can locate a virtual image. The paraxial approximation uses rays close to the axis and neglects thickness and aberrations. With $s>f>0$, $s'$ is positive and M is negative: the image is real and inverted. With $0<s<f$, $s'$ is negative and M is positive: the image is virtual, upright and enlarged. At $s=f$, emerging rays are parallel and no finite image plane exists. A diverging lens with a real object produces an upright reduced virtual image.

## Apply reflected-image geometry

For a spherical mirror in the same real-is-positive convention, concave $f>0$ and convex $f<0$, with $f=R/2$ in the paraxial limit. A real reflected image is in front of the mirror on the incoming-light side; a virtual image is behind it. Thus a positive image distance has a different physical side for a mirror than for a transmitting lens. A parallel ray reflects through the concave focus; a ray aimed through the centre of curvature retraces its path. Convex reflected rays diverge as if from the focus behind the mirror. Use the same equation and $M=-s'/s$ with signed values, rather than silently combining mirror

geometry with a lens coordinate diagram.

## Locate the next element's object

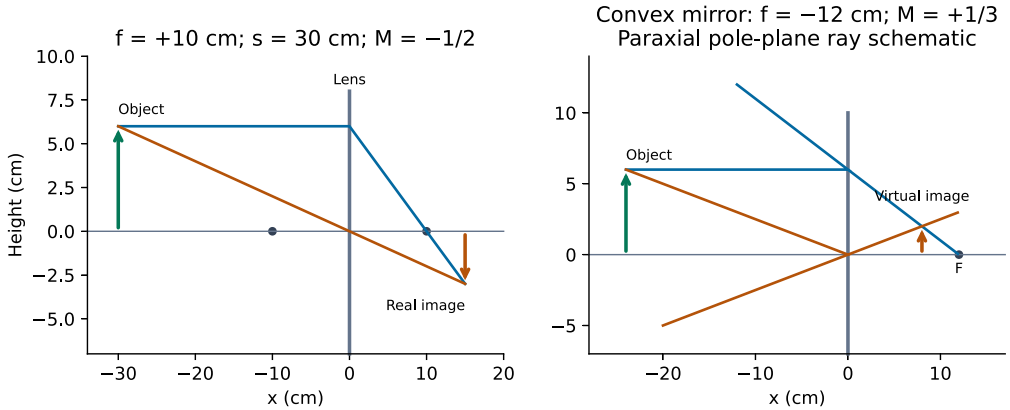
For a sequence of separated lenses, first calculate the actual image coordinate from the first lens. Relative to the next lens, decide whether the incoming rays diverge from a point before it or are still converging toward a point beyond it; these are real and virtual objects respectively. Only then assign its signed object distance and solve again. Total transverse magnification is the product of the individual signed magnifications for aligned paraxial elements. Element separation is not automatically the second object distance. Geometric real/virtual character depends on convergence, not on whether the final image is magnified. These ideal results do not model wave-optical resolution or spherical/chromatic aberration.

## Worked method

Use real-is-positive distances and positive focal length for a converging lens. With f = 12 cm and real object distance s = 8 cm,

$$1/s' = 1/f - 1/s = 1/(12\,\mathrm{cm}) - 1/(8\,\mathrm{cm}) = -1/(24\,\mathrm{cm}).$$

Thus s' = -24 cm and $M = -s'/s = +3$. The virtual image is upright, on the object's side. Signed distances identify the physical side; a negative distance is not an arithmetic failure.




*Thin-lens equation: $s = 30$ cm, $f = 10$ cm gives $s' = 15$ cm and $M = -1/2$*

## Check conditions and vocabulary

Keep signs until the geometry is interpreted; a positive mirror image lies on the incident side, while a positive transmitting-lens image lies on the outgoing side.

**virtual image**: An apparent image located by backwards ray extensions rather than actual outgoing-ray convergence.

**transverse magnification**: Signed image-height to object-height ratio, $-s'/s$ in the declared convention.

# Thermal transport, calorimetry and expansion

GRE Subject Test · GRE Physics

Two wall layers in series carry the same steady heat rate 热流率, but their temperature drops need not be equal.

Prerequisites: 4.

- Combine conductive thermal resistances and distinguish heat rate from heat flux
- Balance sensible and latent heat 潜热 with stated isolation and heat-capacity conditions
- Calculate free thermal expansion and constrained thermal stress with their limits

## Add thermal resistances

For one-dimensional steady conduction through a uniform slab, Fourier's law gives heat flux q_x=−$\kappa$ dT/dx. With constant conductivity $\kappa$, area A, length L and hot-to-cold temperature difference $\Delta$T, heat rate is Qdot=$\kappa$A$\Delta$T/L. Define thermal resistance 热阻 R_th=L/($\kappa$A), measured in K/W; temperature drop is Qdot R_th. Series layers without internal sources carry the same Qdot and their resistances add. Parallel paths at the same endpoint temperatures have heat rates that add, so inverse resistances add. Heat flux is rate per area, W/m$^2$, and need not match between layers of different area. Interface contact resistance and heat leakage must be included if specified.

## Balance sensible and latent energy

Heat capacity C=dQ/dT depends on the thermodynamic path and is not the same as specific heat c per mass. For a material interval without a phase transition, Q=$\int$mc(T)dT, or mc$\Delta$T when c is constant. In an isolated calorimeter, sum the energy changes of all objects, including the container when relevant, and set the sum to zero. At a phase transition at its equilibrium temperature, added energy can change phase fraction rather than temperature: Q=m$\ell$ uses latent heat $\ell$ in J/kg. First supply the sensible heat to reach the transition, then budget latent heat. Do not let a simple weighted-temperature average predict an impossible temperature when melting or freezing is part of the process.

## Check expansion constraints

For a freely expanding rod and small temperature change, $\Delta L \approx \alpha L_0 \Delta T$ with linear expansion coefficient $\alpha$. For an isotropic solid with small strain, each dimension acquires factor $1+\alpha\Delta T$, so $\Delta A/A \approx 2\alpha\Delta T$ and $\Delta V/V \approx 3\alpha\Delta T$. A hole expands with the surrounding material as if its missing region had expanded too. These relations assume a nearly constant coefficient and no mechanical constraint; anisotropic crystals need directional coefficients. If an elastic rod is prevented from changing length, its mechanical strain cancels thermal strain. With tensile stress positive, $\sigma \approx -Y\alpha\Delta T$ during heating, where Y is Young's modulus. This small-strain estimate requires elastic response without yielding or buckling.

## Choose the transport model

Conduction transfers energy through a temperature gradient, convection transports it with moving matter, and thermal radiation can cross a vacuum. For a surface at T facing large surroundings at T_env, a simple grey-body model gives net radiative rate $\varepsilon\sigma$_SB A(T$^4$−T_env$^4$). Use absolute kelvin in these fourth powers, rather than Celsius values. A lumped body whose internal temperature remains nearly uniform can obey C dT/dt=−hA(T−T_env) under Newton cooling with constant h. Then the temperature excess decays with $\tau$=C/(hA), rather than the entire Celsius or kelvin temperature decaying toward zero. This approximation requires sufficiently small internal gradients; a conduction-limited body needs a spatial temperature model.

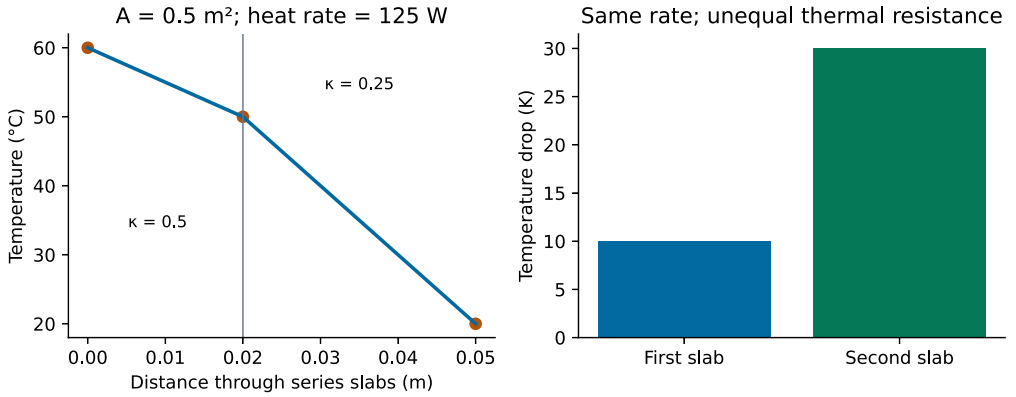
## Worked method

Two slabs in series carry the same steady heat rate. Let resistances be 0.08 and 0.24 K/W. Boundary temperatures are 60 and 20 degrees Celsius.

$$\dot{Q} = \Delta T/(R_1 + R_2) = (40\,\mathrm{K})/(0.32\,\mathrm{K/W}) = 125\,\mathrm{W}.$$

$$\Delta T_1 = \dot{Q} R_1 = (125\,\mathrm{W})(0.08\,\mathrm{K/W}) = 10\,\mathrm{K}.$$

The interface is 50 degrees Celsius when slab 1 touches the hot side. Heat rate has watt units; flux divides it by area.




*The same heat current crosses both slabs; the slab with more thermal resistance takes the larger drop*

## Check conditions and vocabulary

Keep W separate from $W/m^2$, include the calorimeter or latent energy when needed, and do not use free-expansion length together with constrained-stress assumptions.

**thermal resistance**: Temperature difference per steady heat-transfer rate, measured in K/W.

**latent heat**: Energy transferred during a phase change at its transition temperature; specific latent heat is per mass.



# Partition derivatives, energy fluctuations and heat capacity

GRE Subject Test · GRE Physics

A thermal energy gap can keep heat capacity small at both low and high temperatures, even while the excited-state probability keeps increasing.

Prerequisites: 20.

- Obtain canonical mean energy and free energy from a fixed energy spectrum
- Relate energy variance to constant-volume heat capacity with fixed-spectrum conditions
- Evaluate entropy and low/high-temperature limits for an original finite-level model

## Differentiate a fixed spectrum

For a system exchanging energy with a bath at T while N and V remain fixed, let $\beta$=1/(kBT). Sum Z=Σ_i g_i exp(−$\beta$E_i) over energy levels with their degeneracies. Level probability is g_i exp(−$\beta$E_i)/Z; mean energy U is the probability-weighted energy sum. Differentiating a temperature-independent spectrum gives U=−$\partial$lnZ/$\partial\beta$. Keep the energy gap and degeneracies fixed during the derivative. Writing Δ=kBT ln3 to describe one evaluation temperature must not be read as making Δ change with T. For N independent distinguishable identical two-level subsystems, Z_total=z^N and U_total=N u. Indistinguishable particles and interactions need their own state counting rather than this product assumption.

## Identify the thermodynamic quantity

The Helmholtz free energy 亥姆霍兹自由能 is F=−kBT lnZ. Canonical entropy follows from S=kB(lnZ+$\beta$U), equivalently −kBΣ p_j ln p_j over individual microstates. Constant-volume heat capacity is C_V=($\partial$U/$\partial$T)_V,N. These quantities carry different units and describe different derivatives. Adding a constant energy offset $\varepsilon$ to every state multiplies Z by exp(−$\beta\varepsilon$), increases U and F by $\varepsilon$, and leaves probabilities, entropy and heat capacity unchanged when $\varepsilon$ is independent of T. A negative chosen mean energy is therefore not evidence of a negative heat capacity. Use the same energy reference in the probabilities and thermodynamic expressions.

## Relate fluctuations and response

The second $\beta$ derivative of lnZ gives variance Var(E)=⟨$E^2$⟩−$U^2$. For the same fixed-spectrum canonical model, C_V=Var(E)/(kB$T^2$), so C_V/kB=$\beta^2$Var(E). Nonnegative variance implies nonnegative C_V within these conditions. The denominator includes kB, not $kB^2$, when heat capacity retains its ordinary J/K units. This fluctuation formula concerns the canonical energy distribution; it does not say each particle

has exactly the mean energy. For independent subsystems variances add, while means add too. Relative energy fluctuations typically shrink like $1/\sqrt{N}$ when the mean and per-subsystem variance remain finite and nonzero.

## Test finite-level temperature limits

For one ground state at E=0 and one excited state at fixed E=Δ>0, put x=Δ/(kBT). Then z=1+e^(−x), p_exc=1/(1+e^x), u=Δp_exc and C/kB=x²e^x/(1+e^x)². At low T, excitation and heat capacity vanish exponentially. At high T, p_exc tends to one half and energy saturates, so heat capacity tends to zero again. Entropy rises from zero for the unique ground state toward kB ln2 as the two states become equiprobable. The resulting finite-temperature heat-capacity peak is specific to the finite-level model; it is not the constant classical oscillator value. Ground-state degeneracy or additional levels would change the limiting entropy and temperature response.

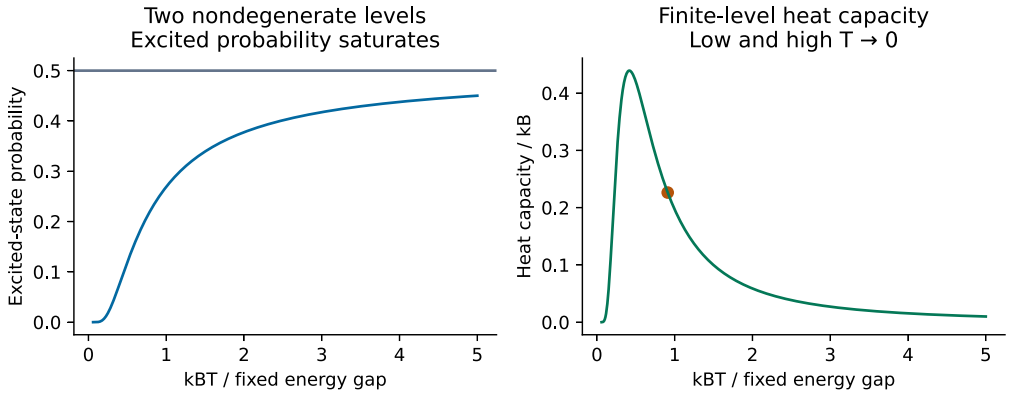
## Worked method

Hold the two energy levels 0 and Delta fixed when differentiating the partition function 配分函数.

$$Z = 1 + e^{-\beta\Delta}, \quad U = -\partial_\beta \ln Z = \frac{\Delta}{1 + e^{\beta\Delta}}.$$

$$\mathrm{Var}(E) = \partial_\beta^2 \ln Z = \frac{\Delta^2 e^{\beta\Delta}}{(1 + e^{\beta\Delta})^2}, \quad C_V = \mathrm{Var}(E)/(k_B T^2).$$

At $\beta\Delta = \ln 3$, excited probability is 1/4, mean energy Delta/4, variance $3\Delta^2/16$ and $C_V/k_B = 3(\ln 3)^2/16 = 0.2263$. Degeneracy, if present, belongs in Z before differentiation.




*A finite level system has a Schottky peak; its heat capacity vanishes at low and high T*

## Check conditions and vocabulary

Hold Δ fixed when differentiating, include degeneracies in Z, and use energy variance rather than the square of mean energy in the heat-capacity formula.

**energy fluctuation** 能量涨落: Canonical spread of energy about its ensemble mean, quantified by the energy variance.

**Helmholtz free energy**: Thermodynamic potential F=U−TS, equal to −kBT lnZ for the canonical ensemble.

# Quantum occupations and limits of equipartition

GRE Subject Test · GRE Physics

## Quantum occupations and limits of equipartition 能量均分

A fermionic energy level can contain several particles when it contains several distinct states; the exclusion rule applies per complete state.

Prerequisites: 20, 35, 38.

- Compute mean occupation 平均占据数 per complete quantum state for fermions and bosons
- Count allowed identical-particle occupations and identify the classical dilute limit
- Compare classical quadratic-mode heat capacity with a quantum oscillator response

## Assign occupation to complete states

For noninteracting particles in thermal and particle exchange equilibrium, use chemical potential $\mu$ and x=($\varepsilon-\mu$)/(kBT). Mean occupation of one complete state is n_F=1/(e^x+1) for fermions and n_B=1/(e^x−1) for bosons. Fermionic occupation of that state is zero or one; its mean lies between them. Bosonic occupation can exceed one. A level of degeneracy g has total mean g times the single-state occupation when its states share the same energy. Count spin as part of a complete state. The Bose denominator requires $\varepsilon>\mu$ for the ordinary finite expression, with the ground-state limit treated separately. For equilibrium photons $\mu$=0 because photon number is not conserved; do not set $\mu$=0 for every material particle gas.

## Count indistinguishable configurations

Count occupation patterns rather than labelling identical particles. Two identical fermions distributed among four distinct complete states have choose(4,2)=6 allowed patterns. Two identical bosons among those states have choose(4+2−1,2)=10 patterns because both may share one state. Two labelled distinguishable particles would have $4^2$=16 assignments. These are different counting models, not three interchangeable answers to the same specification. If a question supplies spin degeneracy, first decide whether its stated number counts complete states or just orbital levels. A Pauli prohibition on two identical complete states does not prohibit opposite-spin fermions in one spatial orbital.

## Check the dilute approximation

When x is large and positive, occupation is small and both denominators are dominated by e^x: n_F≈n_B≈e^(−x), the Maxwell–Boltzmann dilute limit. At x=ln4 the means are 1/5 for fermions, 1/3 for bosons and 1/4 in the classical approximation; the difference is still significant. A large total particle number alone does not justify classical statistics: density, temperature and accessible states control occupation. For fermions at low T, states below $\mu$ become nearly occupied and those above nearly empty. At $\varepsilon=\mu$ a fermionic state has mean one half; inserting that value into the ordinary Bose formula would instead produce a divergent denominator and requires different limiting treatment.

## Test quantum versus classical modes

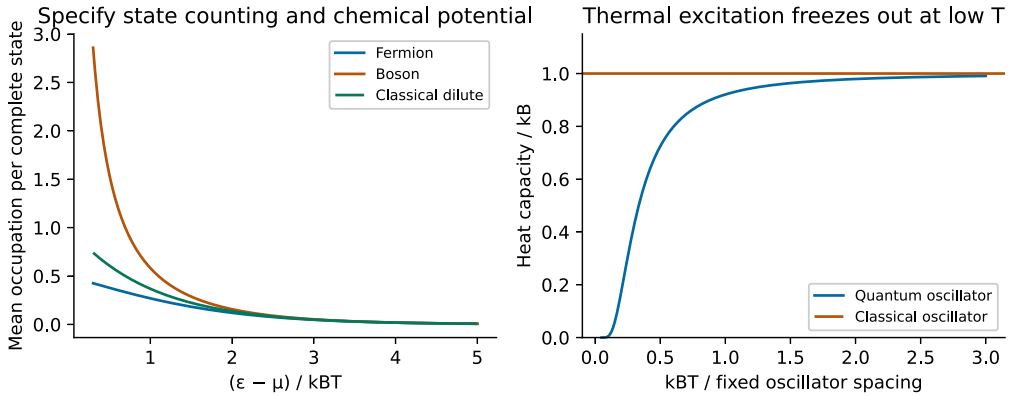
Classical equipartition assigns kBT/2 of mean energy to each independent quadratic term in an equilibrated Hamiltonian. A monatomic ideal gas has three translational terms, giving U=3NkBT/2 and C_V=3NkB/2. One classical one-dimensional harmonic oscillator has kinetic and potential terms, giving mean kBT and C=kB. For a quantum oscillator with fixed spacing $\varepsilon=\hbar\omega$, the thermal energy above its temperature-independent zero point is $\varepsilon$/(e^x−1), now x=$\varepsilon$/(kBT), and C/kB=$x^2$e^x/(e^x−1)$^2$. At high T it approaches the classical value; at low T excitation freezes out and C→0. A zero-point energy $\varepsilon/2$ shifts U but not C. Molecular rotational/vibrational contributions likewise need their energy scales checked before assigning classical quadratic terms.

## Worked method

Occupation number 占据数 refers to one complete single-particle state. For $x = (\epsilon-\mu)/(k_BT) = \ln 4$,

$$\bar{n}_F = (e^x+1)^{-1} = 1/5, \qquad \bar{n}_B = (e^x-1)^{-1} = 1/3.$$

The classical dilute approximation gives $e^{-x} = 1/4$; it is not exact here. A g-fold level has mean total occupation $g\bar{n}$, so a fermionic level can contain more than one particle while each complete state still obeys exclusion.




*All three distributions agree when $(\varepsilon - \mu) \gg kT$; quantum heat capacity freezes out at low $T$*

## Check conditions and vocabulary

Exclude two fermions from one complete state, not from an entire degenerate energy level. State which x is used; chemical-potential occupations and fixed oscillator excitation formulas are different models.

**mean occupation**: Ensemble average number of particles in one complete quantum state or a specified group of states.

**equipartition**: Classical equilibrium rule assigning kBT/2 to each independent quadratic Hamiltonian term.



# Weak perturbations and degenerate subspaces

GRE Subject Test · GRE Physics

A zero first-order energy shift 一阶能量修正 can coexist with a changed state and a nonzero higher-order shift.

Prerequisites: 5, 24.

- Calculate nondegenerate first-order energy shifts as normalised expectation values
- Use symmetry and matrix elements to assess weak-coupling limits
- Diagonalise the perturbation within an exactly degenerate subspace 简并子空间

## Weight the perturbation by probability

Let H=H$_0$+$\lambda$W, where $\lambda$ is a small dimensionless parameter and the eigenstates of H$_0$ are known and normalised. For a nondegenerate level n, the first-order energy correction is $\lambda\langle$n|W|n$\rangle$. In position space this is $\lambda\int\psi$_n*(x)W(x)$\psi$_n(x)dx for a multiplicative potential, with integration over the allowed domain. A potential value at one point is not an expectation value; the state's probability density weights the whole domain. For an infinite well 0<x<L, $\psi$_n=$\sqrt{}$(2/L)sin(n$\pi$x/L). Reflection about its centre makes $\langle$x$\rangle$=L/2, so a weak added potential $\gamma$x has shift $\gamma$L/2 for every nondegenerate well level at first order. $\gamma$ has units of energy per length.

## Use parity carefully

For a perturbation odd about the centre, such as $\gamma$(x−L/2), the probability density of an unperturbed well eigenstate is even, so its diagonal expectation vanishes. A centred quadratic perturbation $\eta$(x−L/2)$^2$ instead gives shift $\eta$L$^2$[1/12−1/(2$\pi^2$n$^2$)], which is positive for $\eta$>0 and depends on n. Parity makes the first-order integral zero only for the specified state and operator symmetry. Off-diagonal matrix elements can still change the wavefunction. Do not turn a symmetry cancellation into a claim that every energy correction vanishes or that the original state remains exact.

## Compare coupling with level gaps

For a nondegenerate state, the leading admixture of another unperturbed state m is proportional to $\lambda$W_mn/(E_n$^0$−E_m$^0$). The useful smallness condition therefore compares coupling matrix elements with the relevant level separations, not just with an arbitrary absolute energy zero. To second order, the energy correction contains $\Sigma$_(m≠n)|$\lambda$W_mn|$^2$/(E_n$^0$−E_m$^0$). For the lowest nondegenerate state, all these denominators are negative, so the second-order correction is nonpositive in this model. Near a degeneracy, a small denominator defeats the nondegenerate expansion; use a coupled subspace instead of dividing by zero.

## Resolve a degenerate subspace

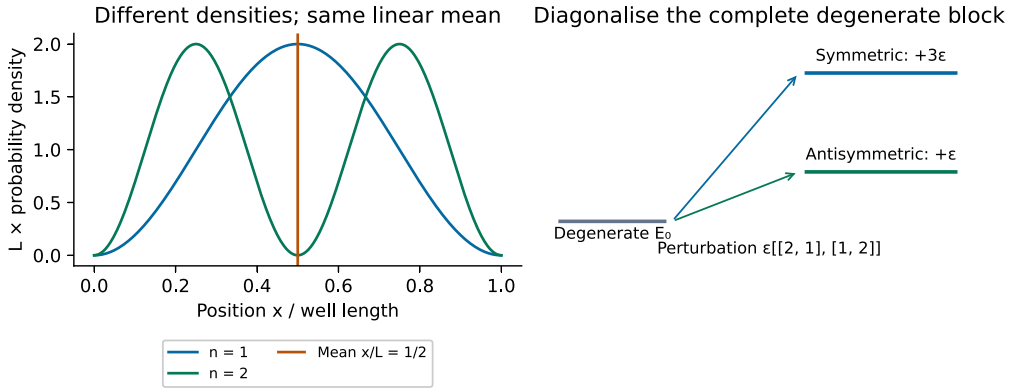
If a level of $H_0$ is exactly degenerate, choose an orthonormal basis within that subspace and form the Hermitian matrix of the perturbation there. Its eigenvalues are the first-order energy shifts, and its eigenvectors specify the combinations that diagonalise the leading splitting. For a two-state subspace with perturbation $\varepsilon[[2,1],[1,2]]$, the normalised symmetric and antisymmetric combinations have shifts $3\varepsilon$ and $\varepsilon$. Reading only the two diagonal entries would incorrectly predict two shifts of $2\varepsilon$. A common scalar multiple of the identity shifts both states equally and does not split their degeneracy; off-subspace couplings can matter at higher order.

## Worked method

A degenerate subspace requires matrix diagonalisation before first-order shifts. Suppose the perturbation restricted to two states is $\epsilon \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.

$$\det(W - wI) = (2\epsilon - w)^2 - \epsilon^2 = 0.$$

Shifts are epsilon and 3 epsilon. Normalised states are $(1,-1)/\sqrt{2}$ and $(1,1)/\sqrt{2}$. Reading only the diagonal entries would miss the splitting. Couplings to other levels must remain weak relative to their energy gaps.




*Diagonalise the whole degenerate block: the eigenvalues give the first-order shifts*

## Check conditions and vocabulary

Use normalised state weights, keep perturbation units, and diagonalise a degenerate block. First-order cancellation and exact invariance are different claims.

**first-order energy shift**: Leading weak-perturbation correction given by the unperturbed state's expectation value.

**degenerate perturbation theory** 简并微扰理论: Method that first diagonalises the perturbation within an unperturbed degenerate subspace.



# Exchange symmetry, spin pairs and Pauli exclusion

GRE Subject Test · GRE Physics

Two electrons can share one spatial orbital if their complete spatial-plus-spin state has the required antisymmetry.

Prerequisites: 5, 36.

- Construct normalised symmetric and antisymmetric 反对称 two-orbital spatial states
- Combine spatial and spin symmetry to produce allowed two-electron states
- Count complete-state occupations and interpret exchange-induced correlations

## Exchange complete particle labels

For identical particles, exchanging every coordinate and spin label changes a fermionic total wavefunction by a minus sign and leaves a bosonic one unchanged. Mathematical slots 1 and 2 label arguments, not permanently distinguishable particles. For two distinct orthonormal orbitals a and b, spatial combinations are Ψ_±=[a(1)b(2)±b(1)a(2)]/√2. Orthogonality makes these combinations normalised and gives exchange eigenvalues ±1. If the two orbitals are identical, the antisymmetric combination is identically zero and the displayed symmetric formula is not correctly normalised; the double-occupation spatial state is simply a(1)a(2). Recheck normalization whenever orbitals or their overlaps change.

## Pair spatial and spin symmetry

Two spin-1/2 particles have one spin singlet 自旋单态 (↑↓−↓↑)/√2 with total spin S=0; it is antisymmetric under exchange. The three triplets ↑↑, (↑↓+↓↑)/√2 and ↓↓ have S=1 and are symmetric. Electrons require an antisymmetric total state, so a symmetric spatial state pairs with the singlet, while an antisymmetric spatial state pairs with a triplet. Two electrons in the same spatial orbital can form the singlet, but cannot form a triplet there in this simple two-electron state. Pauli exclusion prevents occupation of the same complete single-particle state, including spin. Opposite-spin electrons in one orbital are two different complete states; describing exclusion as one electron per orbital discards this distinction.

## Count complete states

For two spatial orbitals a,b and two spin states each, there are four complete single-particle states. Two identical electrons have choose(4,2)=6 occupation patterns: two double-occupation singlets, one different-orbital singlet, and three different-orbital triplets. For three spatial orbitals, there are six complete states and choose(6,2)=15

patterns. Counting ordered assignments overcounts identical particles. Spinless bosons in two orbitals instead have three occupations: both in a, one in each, both in b. These counts assume no additional energy restriction and the stated accessible single-particle states; restricting total energy or spin projection changes the allowed subset.

## Interpret the joint density

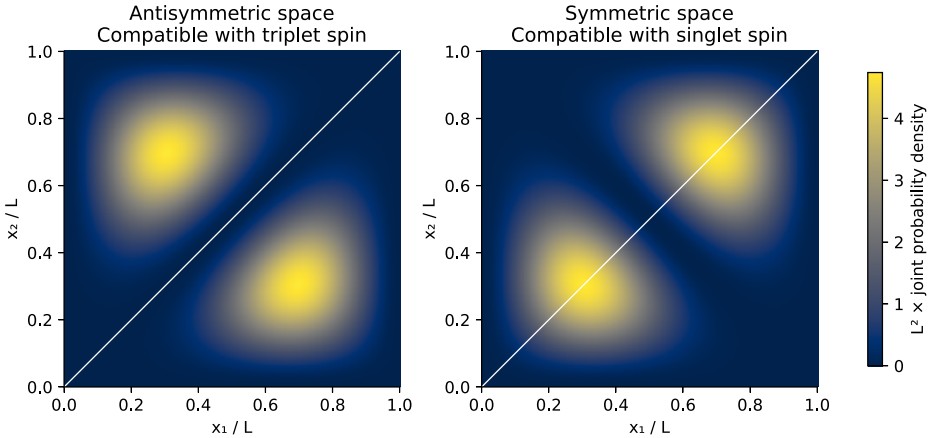
An antisymmetric spatial state satisfies $\Psi_-(x,x)=0$, so its joint position density vanishes on the coincidence line. A symmetric spatial state need not vanish there. The difference arises from interference between exchanged amplitudes, even for noninteracting particles; it is not a separately imposed classical repulsive force. For a well of length L, choose $a(x)=\sqrt{(2/L)}\sin(\pi x/L)$, $b(x)=\sqrt{(2/L)}\sin(2\pi x/L)$. At $x_1=L/4$ and $x_2=3L/4$, the antisymmetric spatial amplitude is $-2/L$ and its density is $4/L^2$; the symmetric amplitude is zero. These are joint probability densities per two lengths, not dimensionless probabilities at exact points. Physical spin compatibility still decides which total electronic state uses each spatial combination.

## Worked method

For orthonormal different orbitals a and b, symmetric and antisymmetric spatial states are

$$\Psi_\pm = [a(1)b(2) \pm b(1)a(2)]/\sqrt{2}.$$

A two-electron total state must be antisymmetric under simultaneous exchange of position and spin. Thus symmetric space pairs with the spin singlet; antisymmetric space pairs with triplet spin. If both electrons occupy a, the spatial product is symmetric and only singlet spin is allowed. The different-orbital normalisation formula cannot simply be reused for a = b.




*An antisymmetric space wavefunction pairs with triplet spin, a symmetric one with singlet spin*



## Check conditions and vocabulary

Exchange all spatial and spin arguments. Do not confuse antisymmetric space with antisymmetric total state, or use the distinct-orbital $\sqrt{2}$ factor for two identical orbitals.

**spin singlet**: Antisymmetric two-spin-1/2 state with total spin S=0.

**Slater determinant** 斯莱特行列式: Antisymmetric fermionic construction from complete single-particle states; duplicate states make it vanish.

# Finite-well bound states and hydrogenic quantum numbers

GRE Subject Test · GRE Physics

A bound wavefunction can extend outside a finite well 有限深势阱 even though the particle's energy is below the external potential.

Prerequisites: 5, 24, 37.

- Match decaying finite-well states and derive parity-dependent quantisation conditions
- Interpret dimensionless bound-state roots and distinguish confinement from scattering
- Use hydrogenic quantum numbers, angular momentum and radial probability measures

## Match bound-state asymptotes

Declare a symmetric well $V=-V_0$ for $|x|<a$ and $V=0$ outside, with $V_0>0$ and the same mass throughout. A bound energy lies between $-V_0$ and zero. Inside, $k=\sqrt{[2m(E+V_0)]}/\hbar$ gives oscillatory solutions; outside, $\kappa=\sqrt{(-2mE)}/\hbar$ gives decaying tails. Reject growing exponentials to obtain a normalisable state. Reflection symmetry permits even interior cos(kx) or odd sin(kx) states. At finite boundaries without delta interactions, both $\psi$ and $\psi'$ are continuous. Even matching gives $k\tan(ka)=\kappa$; odd matching gives $-k\cot(ka)=\kappa$. A finite well does not require $\psi$ to vanish at its edges as an infinite wall does.

## Solve a dimensionless root

Introduce $z=ka$ and $\rho=a\sqrt{(2mV_0)}/\hbar$. Then $\kappa a=\sqrt{(\rho^2-z^2)}$ and $0<z<\rho$. Solve $z\tan z=\sqrt{(\rho^2-z^2)}$ for an even state, or $-z\cot z=\sqrt{(\rho^2-z^2)}$ for an odd one, on intervals with the appropriate sign and away from tangent poles. Energy follows as $E/V_0=z^2/\rho^2-1$. For $\rho=1$, the ground root lies in $0<z<1$ and the even equation is equivalent there to $z=\cos z$. Bisection or a converged root finder gives $z\approx0.739085$ and $E/V_0\approx-0.453753$. Since $\rho<\pi/2$, no odd bound-state branch fits. In one dimension an attractive square well has an even bound ground state however shallow it is; an $E=0$ threshold tail is not square integrable.

## Normalise the interior and tails

For an even bound state 束缚态 write $\psi=A\cos(kx)$ inside and $\psi=A\cos(ka)\exp[-\kappa(|x|-a)]$ outside. Continuity sets the relative tail amplitude; integrate $|\psi|^2$ over both the interior and tails to determine A. Probability outside the well is nonzero and depends on its depth, width and the selected state. Bound states have discrete energies and normalisable decaying asymptotes. A scattering state with $E>0$ has propagating external waves and is normalised or interpreted using a continuum/flux convention. A decaying tail is not evidence that the state violates energy conservation, and raw tail amplitude is not the outside probability until it has been integrated and normalised.

## Choose the hydrogenic probability measure

For a one-electron Coulomb ion with nuclear charge Ze, use the nonrelativistic central-potential model and a heavy nucleus approximation unless reduced mass is specified. $E_n\approx-13.6\ Z^2/n^2$ eV; allowed orbital numbers are $n\geq1$, $l=0,\ldots,n-1$ and $m_l=-l,\ldots,l$. $L^2$ has eigenvalue $l(l+1)\hbar^2$ and $L_z=m_l\hbar$; the ground 1s orbital has $l=0$, despite the historical Bohr circular-orbit rule. Spatial degeneracy at fixed n is $\Sigma_l(2l+1)=n^2$ before spin and fine-structure corrections. If $\psi=R_{nl}(r)Y_{lm}$ with angular part normalised, radial probability in dr is $r^2|R|^2dr$, not just $|R|^2dr$. For hydrogen 1s, the radial density is $4r^2e^{(-2r/a_0)}/a_0^3$: it peaks at $r=a_0$ and has mean $3a_0/2$, while the three-dimensional point density is largest at the origin. Distinguish these measures before locating a most probable radius.

## Worked method

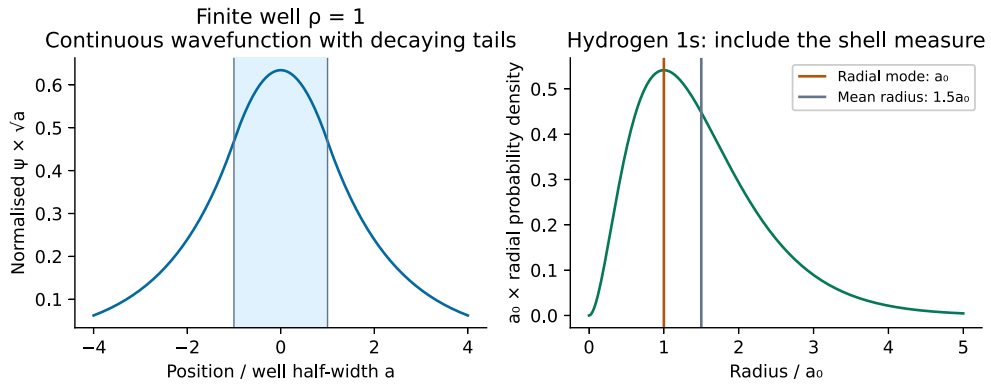
A finite well has V = -V0 for $|x| < a$ and zero outside. A bound energy lies strictly between -V0 and zero. Define

$$k = \sqrt{2m(E + V_0)}/\hbar, \qquad \kappa = \sqrt{-2mE}/\hbar.$$

Even parity gives an interior cosine and decaying tails. Matching value and derivative at a gives

$$k\tan(ka) = \kappa.$$

Odd parity instead gives $-k\cot(ka) = \kappa$. Solve only within valid parity intervals; a root at a tangent pole is not a bound state.




*Include the $r^2$ shell factor: the most probable radius is $a_0$, the mean $1.5a_0$*

## Check conditions and vocabulary

Use the declared potential zero, match $\psi'$ as well as $\psi$, and include the radial shell measure. A bound tail and a continuum travelling wave have different energy regimes.

**bound state**: Normalisable stationary state with confined probability and discrete energy in the specified model.

**radial probability density** 径向概率密度: Probability per radial distance, including the spherical-shell measure after angular integration.



# Atomic field shifts and spectral differences

GRE Subject Test · GRE Physics

The same magnetic field needs different quantum labels when its energy scale crosses the spin-orbit splitting.

Prerequisites: 7, 37.

- Calculate orbital and spin magnetic shifts in the appropriate coupling regime
- Find photon-energy shifts from allowed upper-minus-lower level changes
- Contrast parity cancellation with degenerate electric-field splitting

## Choose magnetic quantum labels

An electron has a negative magnetic moment: $\mu$_L=$-\mu$_B L/$\hbar$ and $\mu$_S$\approx-2\mu$_B S/$\hbar$. With B along positive z, the interaction $-\mu \cdot$ B therefore gives H_Z=$\mu$_B B(L_z+2S_z)/$\hbar$. $\mu$_B$\approx5.788\times10^{-5}$ eV/T. In an uncoupled basis with orbital projection m_l and spin projection m_s=$\pm1/2$, the shift is $\mu$_B B(m_l+2m_s). Use this uncoupled rule when the magnetic interaction is large relative to the relevant spin-orbit splitting but still small relative to orbital energy gaps, so quadratic diamagnetic and orbital restructuring effects can be neglected. Large and small are comparisons of energies, not universal Tesla thresholds. In the opposite weak-field regime, m_l and m_s are not independently conserved labels for an LS-coupled eigenstate.

## Resolve a weak-field multiplet

When LS coupling dominates a weak magnetic perturbation, J=L+S and m_J label the multiplet. The first-order shift is $\mu$_B B g_J m_J, with g_J=1+[J(J+1)+S(S+1)$-$L(L+1)]/[2J(J+1)] in the approximation g_s=2. L,S,J here are dimensionless quantum numbers. For L=1,S=1/2, J=3/2 gives g_J=4/3 and shifts $-2,-2/3,+2/3,+2$ times $\mu$_B B. J=1/2 gives g_J=2/3 and shifts $\pm\mu$_B B/3. Do not substitute J=0 into the singular formula; a J=0 state has no first-order vector projection shift. Intermediate fields require a coupled Hamiltonian rather than mixing weak-field and uncoupled formulas.

## Subtract the two level shifts

A photon's energy is the upper atomic energy minus the lower energy. Its field-induced change is therefore $\Delta$E_$\gamma$=$\Delta$E_upper$-\Delta$E_lower. Count allowed transitions using the stated selection rules as well as the two level patterns. Several transitions can share one frequency; the number of split states is not automatically the number of spectral lines. In a spin-neglected orbital model, an upper L=1 triplet has shifts m_l $\mu$_B B with m_l=$-1,0,+1$, and a lower L=0 state has zero shift. Electric-dipole $\Delta$m_l=$0,\pm1$ permits three photon shifts $-\mu$_B B,0,$+\mu$_B B. This normal-triplet model does not describe every real atom or fine-structure multiplet. Polarisation and

observation direction can change which components are detected.

## Use electric-field symmetry and degeneracy

In a uniform electric field F along z, an electron's perturbation is +eFz for the stated electrostatic potential convention. A nondegenerate parity eigenstate has ⟨z⟩=0, so its first-order diagonal Stark shift vanishes, although a quadratic shift can remain. Exactly degenerate opposite-parity states instead allow an off-diagonal dipole matrix element. In an original two-state model, suppose the dipole matrix is [[0,d],[d,0]], with real d in charge×length units. The perturbation F times this matrix has eigenvalues ±dF and opposite superpositions of the two states. Changing a basis phase changes the off-diagonal sign but not the splitting 2|dF|. Actual near-degenerate atoms require comparison with fine structure and other small splittings before an exactly degenerate approximation is adopted; this supplied matrix is not a universal hydrogen coefficient.

## Worked method

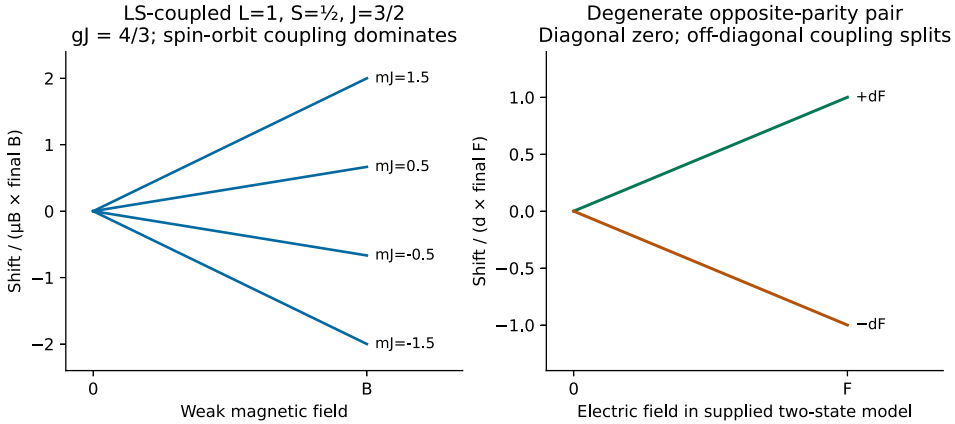
In weak LS-coupled magnetic fields, the Zeeman effect 塞曼效应 uses J, not uncoupled ml and ms.

$$\Delta E = \mu_B B g_J m_J, \quad g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

For L = 1, S = 1/2 and J = 3/2, $g_J = 4/3$.

$$\Delta E = \mu_B B g_J m_J = (5.788 \times 10^{-5}\,\mathrm{eV/T})(2\,\mathrm{T})(4/3)(1/2) = 7.72 \times 10^{-5}\,\mathrm{eV}.$$

Select the field regime before inserting quantum numbers. Photon shifts subtract the lower-level shift.




*Weak-field Zeeman shifts are gJ · mJ · μB · B; an electric field splits a degenerate opposite-parity pair linearly*

## Check conditions and vocabulary

Declare the coupling regime, keep the negative electron moment sign, and subtract lower-level shifts. Zero diagonal dipole expectation does not exclude degenerate mixing or quadratic shifts.

**Landé factor** 朗德因子: Projection factor relating a weak-field LS-coupled magnetic shift to g_J m_J.

**Stark splitting** 斯塔克分裂: Electric-field-induced separation of atomic energy levels, with symmetry and degeneracy controlling leading order.

# Thermal spectra and one-electron scaling

GRE Subject Test · GRE Physics

A spectral peak, a total radiation flux and an atomic transition energy are three different observables.

Prerequisites: 7, 9, 34.

- Distinguish wavelength spectral peak, integrated radiation and photon energy
- Derive Bohr radius and energy scaling under one-electron assumptions
- Calculate Coulomb-ion transitions, series limits and ionisation thresholds

## Identify the spectral measure

For ideal thermal equilibrium radiation, Planck's wavelength spectral radiance 光谱辐亮度 is B_λ=2hc²/{λ⁵[exp(hc/(λk_B T))−1]}. B_λ is per wavelength interval and per solid angle, not a total power. A blackbody's hemispheric surface flux spectrum is M_λ=πB_λ; integrating over all wavelengths gives σ_SB T⁴. All temperatures are absolute Kelvin. The wavelength peak obeys λ_max T≈2.898×10⁻³ m·K. A spectrum per frequency has a different peak because B_ν dν and B_λ dλ include a Jacobian; c/λ_max is not the peak frequency of B_ν. Increasing T moves the wavelength peak shorter and increases the integrated flux by T⁴, not by the peak-position ratio alone.

## Separate power from one photon

For an original uniform grey surface of area A and wavelength-independent emissivity ε, net radiative power to a large uniform environment is εσ_SB A(T⁴−T_env⁴) under the stated view-factor assumptions. Use σ_SB≈5.670×10⁻⁸ W·m⁻²·K⁻⁴. Spectrally varying emissivity or incomplete surroundings requires a more detailed model. A photon at a specified wavelength has E=hc/λ, conveniently about 1240 eV·nm/λ_nm. A thermal spectrum contains many photon energies; a photon at the wavelength peak is not the mean energy of every photon. For T=3000 K, λ_max≈966 nm; its photon energy is about 1.284 eV. A surface with A=2×10⁻⁴ m² and ε=0.5 emits 459.27 W to a negligibly cold environment.

## Derive one-electron scaling

In the historical Bohr one-electron model with a heavy nucleus of charge Ze, Coulomb force m_e v²/r=Ze²/(4πε₀r²) and angular momentum m_e vr=nℏ lead to r_n=a₀n²/Z, v_n=Zαc/n and E_n≈−13.6Z²/n² eV. The radius scales with n²/Z while binding energy scales with Z²/n²; do not use the same charge power in both. These circular-orbit assumptions are a historical scaling model. Wave-mechanical angular momentum is √[l(l+1)]ℏ with l=0,…,n−1, so a 1s orbital has zero orbital angular momentum rather than the Bohr nℏ value. Reduced-mass corrections replace m_e with μ: Coulomb radius scales as 1/μ and energy as μ. Multielectron screening, large-Z relativistic corrections and fine structure are outside the simple model.

## Subtract levels and locate limits

For an emission ni→nf with ni>nf, E_γ≈13.6Z²(1/nf²−1/ni²) eV. Convert this positive difference to λ≈1240/E_γ nm. Ionisation from n requires energy 13.6Z²/n² eV to reach the continuum zero. Absorption reverses the level ordering and needs the corresponding positive incoming photon energy. For a series ending at fixed nf, the largest bound-bound photon energy occurs as ni→∞, giving E_limit=13.6Z²/nf² and the shortest series wavelength. For one-electron helium Z=2, n=3→2 gives 7.5556 eV and λ≈164.12 nm. Its Bohr n=3 radius is 4.5a₀; this radius describes the historical orbit, not a universal radial mode for every l at n=3.

## Worked method

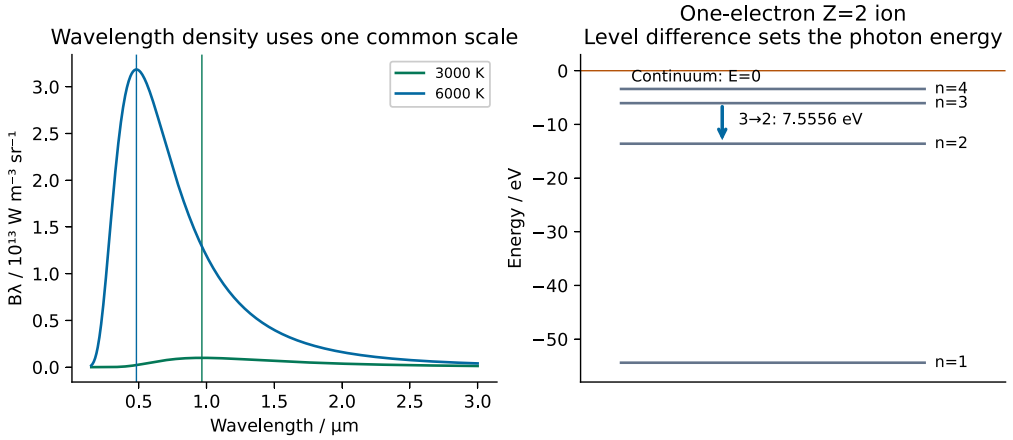
Wien's law 维恩定律 refers to the wavelength-density peak of a blackbody spectrum.

$$\lambda_{max} = b/T = (2.898 \times 10^{-3}\,\mathrm{m\,K})/(3000\,\mathrm{K}) = 966\,\mathrm{nm}.$$

A photon at that wavelength has energy

$$E_\gamma = hc/\lambda = (1240\,\mathrm{eV\,nm})/(966\,\mathrm{nm}) = 1.28\,\mathrm{eV}.$$

The source emits a distribution of photon energies. Its integrated surface flux is $\sigma T^4$, and the frequency-density peak is not obtained by simply using c divided by this peak wavelength.




*Doubling the temperature halves the peak wavelength; for a $Z = 2$ ion every level is $Z^2 = 4$ times the hydrogen value*

## Check conditions and vocabulary

Declare a wavelength or frequency density, use Kelvin for fourth powers, and distinguish transitions from ionisation. A Bohr circular radius and angular momentum are not every wave orbital's radius and L.

**spectral radiance**: Radiation intensity per projected area, solid angle and stated spectral interval.

**series limit** 谱线系限: Limiting bound-bound photon energy or wavelength as the initial level approaches the continuum for a fixed final level.



# Simultaneity, length measurement and velocity transformation

GRE Subject Test · GRE Physics

Two observers can disagree about which of two events happened first, and both can be right.

Prerequisites: 22.

- Transform event separations and identify proper length 固有长度 versus simultaneous endpoint readings
- Apply relativistic velocity addition and check a four-vector invariant
- Classify the interval sign to order events and rule out causal contact

## Transform separations

Use the Lorentz transformation with declared frame motion: if frame S′ moves at +v along x relative to S, then $x'=\gamma(x-vt)$ and $t'=\gamma(t-vx/c^2)$ with $\gamma=1/\mathrm{sqrt}(1-v^2/c^2)$. Apply it to event separations $\Delta x$ and $\Delta t$, not only to single coordinates. Two events simultaneous in S ($\Delta t=0$) at different places have $\Delta t'=-\gamma v\Delta x/c^2$: the event further in the +x direction happens earlier in S′. Simultaneity is frame-dependent, not a bookkeeping error.

## Measure length in one frame

A moving rod length must be read from endpoint positions at the same lab time ($\Delta t=0$). Then $\Delta x'=\gamma\Delta x$ gives $L=L0/\gamma$ with L0 the rest-frame (proper) length. Reading the endpoints simultaneously in the rod frame instead does not produce the lab length; the two reading conditions answer different questions. State the frame and the reading condition before quoting any length.

## Classify the interval

The interval $s^2=c^2\Delta t^2-\Delta x^2$ (mostly-minus convention, declared once) is invariant. $s^2>0$ is timelike: a frame exists with the events co-located, and $\Delta\tau=s/c$ is the proper time between them. $s^2<0$ is spacelike: a frame exists with the events simultaneous, and $\mathrm{sqrt}(-s^2)$ is the proper distance. $s^2=0$ is lightlike. Timelike order is the same in every frame; spacelike order can swap, and no signal can join the events because $|\Delta x/\Delta t|>c$.

## Add velocities and check invariants

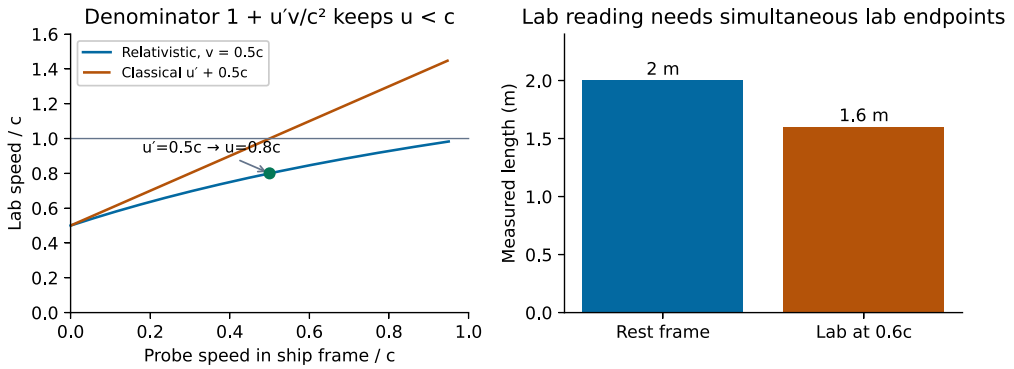
Collinear velocities add as u=(u′+v)/(1+u′v/c²); the denominator keeps u<c for any u′,v<c, and u′=c gives u=c exactly. Energy and momentum form a four-vector with invariant E²−p²c²=m²c⁴: compute it in any convenient frame. Check each result in the nonrelativistic limit, where the addition rule reduces to u=u′+v and the invariant reduces to the rest energy.

## Worked method

Two lab events have Delta x = 900 m and Delta t = 2 microseconds. Their spacetime interval 时空间隔 is spacelike because c Delta t = 600 m is less than Delta x. A frame making them simultaneous satisfies

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \quad v = c^2\Delta t/\Delta x = 2c/3.$$

Their separation in that frame is $\sqrt{\Delta x^2 - c^2\Delta t^2} = 670.8$ m. No frame can put these spacelike events at the same position.




*Velocity addition keeps u below c (0.5c with 0.5c gives 0.8c); length contracts by 1/γ*

## Check conditions and vocabulary

Quoting L=L0/γ without the simultaneous-endpoint condition, or adding velocities as u=u′+v at relativistic speeds. A simultaneity difference is physical, not a mistake.

**spacetime interval**: The invariant combination c²Δt²−Δx² under the declared sign convention, whose sign classifies an event separation.

**proper length**: The endpoint separation measured in the object's rest frame; any other frame must read both endpoints simultaneously in its own time.



# Measurement loading, amplifier gain and detector response

GRE Subject Test · GRE Physics

A voltmeter 电压表 changes the circuit it measures; the reading is a divider output, not the open-circuit voltage.

Prerequisites: 14, 21, 30.

- Calculate measurement loading from finite meter and source impedances
- Determine simple amplifier gain and first-order filter response
- Interpret detector efficiency, resolution, dead time 死时间 and calibration bias

## Model the measurement

Model the measured circuit as a Thevenin source V_th with resistance R_th and the instrument as input resistance R_m. The reading is V_th · R_m/(R_m+R_th), with the same sign as V_th and magnitude no greater than |V_th| for positive resistances. A 10 V source with R_th=100 kΩ read by a 1 MΩ meter gives 10×(1/1.1)=9.09 V, a 9% low bias. A 10× oscilloscope probe raises the effective input resistance and divides the signal by ten; quote both effects.

## Set gain and cutoff

An ideal non-inverting op-amp stage has gain 1+R2/R1; the inverting stage has −R2/R1, with the sign carried explicitly. A first-order RC low-pass has cutoff f_c=1/(2πRC); its amplitude ratio is 1/sqrt(1+(f/f_c)²) and its phase is −arctan(f/f_c). At f=f_c the amplitude is $1/\sqrt{2}$ (the −3 dB point), not one half. For R=1 kΩ and C=100 nF, f_c≈1.59 kHz.

## Rate the detector

Detector efficiency ε is detected events divided by incident events; energy resolution is quoted as FWHM/E, such as 13.2 keV on a 662 keV line, about 2%. A non-paralyzable detector with dead time τ records m=n/(1+nτ) from true rate n; inverting gives n=m/(1−mτ), which fails as mτ→1. State which dead-time model you assume; paralyzable behaviour differs.

## Calibrate with its limits

A linear calibration against known standards can correct offset and scale error; it does not remove noise or guarantee correction of nonlinearity. A calibration curve maps indicated to true values; interpolation between points assumes local smoothness. Report resolution and efficiency separately: sharp peaks with poor efficiency still miss events, and high efficiency with broad resolution still mixes nearby lines.

## Worked method

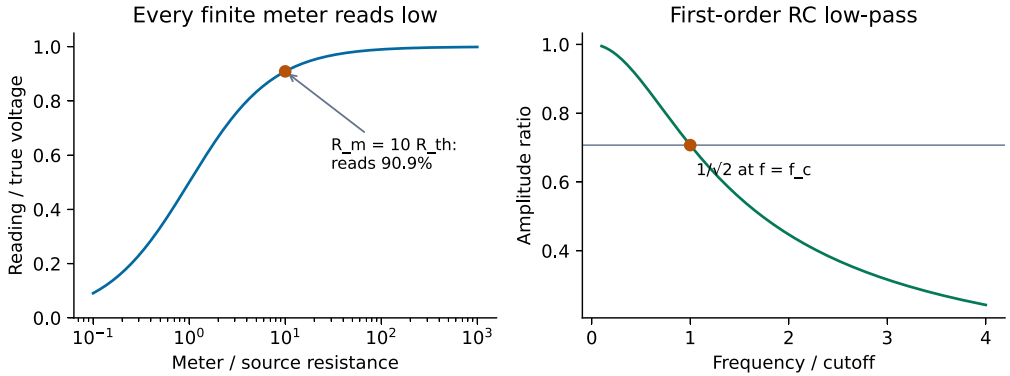
A voltmeter of finite input resistance forms a divider with the source resistance.

$$V_m = V_{th}\frac{R_m}{R_m + R_{th}} = (12\,\mathrm{V})\frac{1.0\,\mathrm{M\Omega}}{1.0\,\mathrm{M\Omega} + 0.20\,\mathrm{M\Omega}} = 10.0\,\mathrm{V}.$$

For a nonparalysable detector with observed rate m and dead time tau,

$$n = m/(1 - m\tau), \qquad m\tau < 1.$$

Efficiency, dead time and spectral resolution are different corrections; one cannot replace the others.




*A finite meter resistance always reads low; a low-pass filter passes 1/√2 at f_c*

## Check conditions and vocabulary

Treating the $1/\sqrt{2}$ cutoff as one half, ignoring that a finite meter reads low through the divider, or applying the non-paralyzable correction beyond its range.

**dead time**: The minimum interval after one recorded event during which the detector cannot record another.

**input impedance** 输入阻抗: The effective load a measuring instrument presents to the circuit under test.



# Radiation attenuation, laser gain and cavity modes

GRE Subject Test · GRE Physics

## Radiation attenuation 衰减, laser gain and cavity modes

A photon beam has no definite range, yet a heavy charged particle does; shielding calculations fail if the two are swapped.

Prerequisites: 7, 17, 21.

- Distinguish charged-particle stopping and range from exponential photon attenuation
- Compute attenuated intensity and detected counts with stated coefficients
- Explain stimulated emission, population inversion 粒子数反转 and optical cavity resonance conditions

## Choose the interaction model

Heavy charged particles lose energy continuously through many small collisions, so they have an approximate range: under a constant-loss model a particle with initial energy E0 and loss rate dE/dx stops after E0/(dE/dx). Electrons straggle more and radiate. Photons instead interact in single events, so a narrow beam attenuates exponentially as I=I0 e^(−μx) with no definite maximum depth; the half-value layer 半值层 is ln2/μ.

## Attenuate and count

Apply the exponential only to the stated narrow-beam geometry: μ=0.2 /cm gives a half-value layer of 3.47 cm and I/I0=e^(−2)≈0.135 after 10 cm. Multiply by detector efficiency for recorded counts: with μx=ln4 and ε=0.25 the recorded fraction is 0.25×1/4=1/16. Build-up from scattered photons makes broad-beam shielding transmit more than the narrow-beam exponential; state the geometry.

## Invert the population

Stimulated emission produces a photon matching the stimulating photon in frequency, direction and phase, giving coherent amplification. It competes with absorption; net gain needs population inversion N2/g2>N1/g1 (N2>N1 for equal degeneracies). Positive-temperature equilibrium has N2/g2<N1/g1 by the Boltzmann factor, so a two-level system in equilibrium cannot lase continuously; practical lasers pump a third level or a metastable state.

## Resonate in the cavity

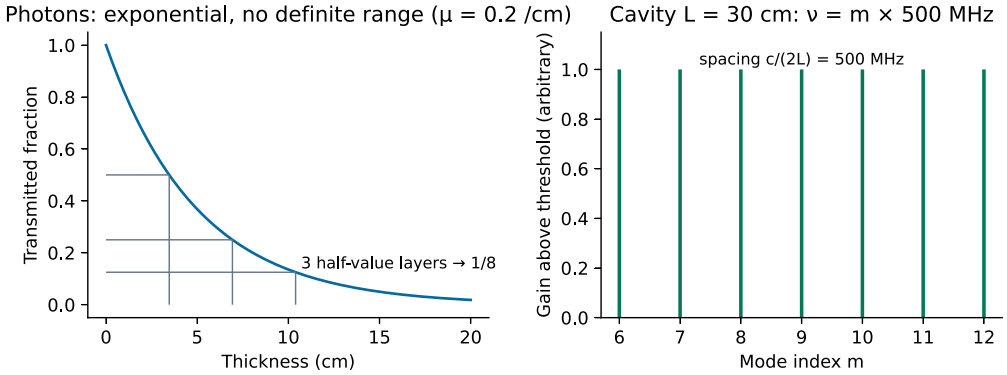
A linear cavity of length L supports standing modes with L=mλ/2, i.e. frequencies ν_m=mc/(2L) with spacing c/(2L): for L=30 cm the spacing is 500 MHz. Gain must exceed the round-trip losses for oscillation. Interferometers such as Michelson use the same coherence: fringe counts track optical path changes, as in the gas-cell measurement.

## Worked method

A narrow beam has attenuation $I = I_0 e^{-\mu x}$ and half-value layer $x_{1/2} = \ln 2/\mu$.

$$x_{1/2} = (\ln 2)/(0.20\,\mathrm{cm}^{-1}) = 3.47\,\mathrm{cm}.$$

Three half-value layers transmit 1/8. This model excludes scattered build-up into the detector. For a linear vacuum laser cavity, adjacent longitudinal modes are separated by $c/(2L)$. Net stimulated gain requires $N_2/g_2 > N_1/g_1$; the simpler $N_2 > N_1$ assumes equal degeneracies.




*Photons have no definite range; cavity modes are spaced c/(2L)*

## Check conditions and vocabulary

Assigning photons a definite range or charged particles a single exponential law, and forgetting that equilibrium two-level populations cannot invert. Check which interaction model the beam species requires.

**half-value layer**: The material thickness that halves an exponentially attenuated beam, equal to $\ln 2/\mu$.

**population inversion**: A non-equilibrium state with more population in the upper laser level than the lower, enabling net stimulated emission.



# Bragg diffraction and electron-gas heat capacity

GRE Subject Test · GRE Physics

## Bragg diffraction 布拉格衍射 and electron-gas heat capacity

Rotating a crystal in an X-ray beam maps its atomic planes, because constructive reflection happens only at discrete angles.

Prerequisites: 8, 36.

- Apply the Bragg condition 布拉格条件 to find plane spacings or diffraction angles
- Estimate free-electron Fermi energy 费米能 and Fermi temperature
- Distinguish electronic and lattice heat-capacity contributions

## Apply the Bragg condition

Bragg reflection from parallel planes spaced d interferes constructively when 2d sinθ=nλ, with θ measured from the plane surface (the glancing angle), not from the normal. First order n=1 gives the smallest angle. For λ=0.154 nm and a peak at θ=20°, d=0.154/(2 sin20°)≈0.225 nm. If 2d<λ no order exists; the sine must stay at or below 1.

## Read the lattice constant

In a cubic crystal the (hkl) plane spacing is d=a/sqrt(h²+k²+l²) with lattice constant a; the (110) planes give a/√2. A measured Bragg angle therefore measures the lattice constant. Systematic absences from the basis (for example body-centred lattices missing h+k+l odd) carry structure information beyond the spacing formula.

## Fill the electron sea

The free-electron model packs N conduction electrons into states up to the Fermi wavevector k_F=(3π²n)^(1/3) for number density n, giving E_F=ħ²k_F²/(2m). For n=8.5×10²⁸ /m³, k_F≈1.36×10¹⁰ /m, E_F≈7.05 eV and T_F=E_F/k_B≈8.2×10⁴ K. Since T_F far exceeds room temperature, only a fraction of order T/T_F of the electrons are thermally excited.

## Separate the heat capacities

Metals therefore have two low-temperature heat-capacity terms: an electronic term γT linear in T and a lattice (Debye) term βT³. Plotting C/T against T² gives intercept γ and slope β. Near room temperature the lattice part nears the Dulong–Petit value and dominates; at low T the γT electronic term beats the collapsing βT³ lattice term.

## Worked method

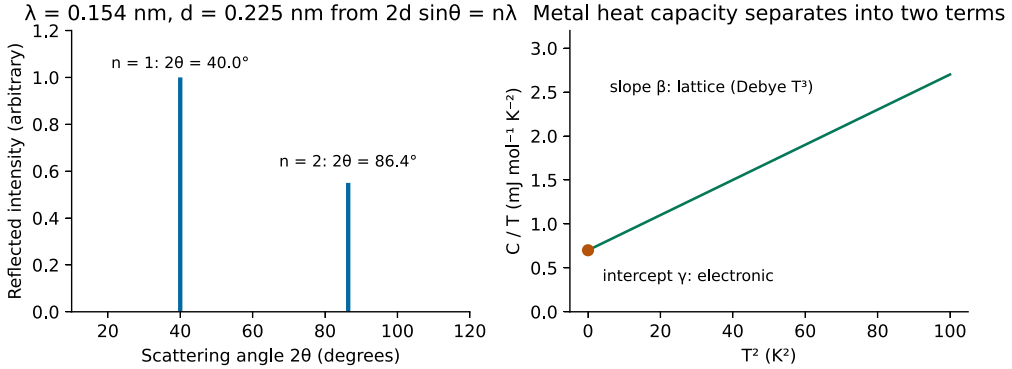
Bragg diffraction uses angle theta from the crystal plane, not from its normal.

$$2d\sin\theta = n\lambda, \qquad d = \lambda/(2\sin\theta) = (0.154\,\text{nm})/[2\sin(20^\circ)] = 0.225\,\text{nm}.$$

Low-temperature metal heat capacity has separate electronic and lattice terms:

$$C = \gamma T + \beta T^3, \qquad C/T = \gamma + \beta T^2.$$

A plot of C/T against T squared has intercept gamma and slope beta. A straight line identifies this model's regime.




*Bragg's law $2d\sin\theta = n\lambda$; $C/T = \gamma + \beta T^2$ separates electrons from lattice*

## Check conditions and vocabulary

Measuring $\theta$ from the plane normal instead of the plane, or assigning the whole low-T heat capacity to electrons. The sine condition and the two-term decomposition are where these calculations fail.

**Bragg condition**: Constructive reflection from lattice planes when 2d sin$\theta$=n$\lambda$, with $\theta$ measured from the plane.

**Fermi energy**: The highest occupied electron energy at zero temperature in the free-electron model.



# Reaction energies, activity and decay balances

GRE Subject Test · GRE Physics

## Reaction energies, activity 活度 and decay balances

One atomic mass unit is 931.5 MeV, so a reaction energy is a small difference between large masses.

Prerequisites: 8, 25, 21.

- Compute reaction energy from complete atomic or nuclear mass bookkeeping
- Calculate activity from half-life and a simple decay-chain balance
- Compare fission and fusion binding changes with charge and nucleon conservation

## Bookkeep the masses

The reaction energy is Q=(m_initial−m_final)c$^2$; Q>0 releases kinetic energy. Track every species, and keep the mass convention consistent: tabulated atomic masses include electrons. For $\beta-$ decay and for $\alpha$ decay with a neutral helium product, atomic electron counts balance directly; $\beta+$ or electron capture needs explicit electron-mass terms. Use 1 u=931.5 MeV/c$^2$.

## Compute Q and check balance

Alpha decay of $^{238}$U with atomic masses 238.0508 u → 234.0436 u + 4.0026 u gives $\Delta$m=0.0046 u and Q≈4.28 MeV. Fusion D+T with 2.0141+3.0161−4.0026−1.0087=0.0189 u gives Q≈17.6 MeV. A reaction also requires charge and nucleon-number balance: $^{14}$N+$\alpha\rightarrow{}^{17}$O+p balances 7+2=9 and 14+4=18.

## Convert half-life to activity

Activity is A=$\lambda$N with decay constant $\lambda$=ln2/T½; one becquerel is one decay per second. For T½=2 h and N=6×10$^{12}$, A=6×10$^{12}$×ln2/7200≈5.8×10$^8$ Bq. After three half-lives both N and A fall by 1/8. Equal activities do not imply equal atom counts: shorter half-life at fixed N gives larger A.

## Compare binding changes

In a chain A→B→C with a long-lived parent ($\lambda$_A≪$\lambda$_B), B accumulates until its activity approaches the parent activity (secular equilibrium); write the balance dN_B/dt=$\lambda$_A N_A−$\lambda$_B N_B. The binding-energy-per-nucleon curve peaks near iron, so heavy fission (about 7.6 to 8.5 MeV per nucleon over roughly 240 nucleons, of order 200 MeV) and light fusion both release energy; compare total binding, not

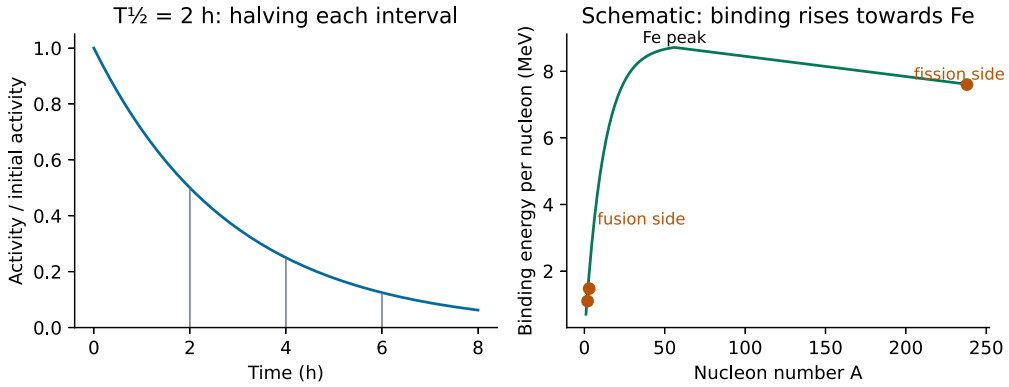
nucleon counts.

## Worked method

Activity is $A = \lambda N$, with $\lambda = \ln 2 / T_{1/2}$. For N = $6.0 \times 10^{12}$ and half-life 2 hours, convert time before calculating.

$$A = (\ln 2)N/T_{1/2} = (\ln 2)(6.0 \times 10^{12})/(7200\,\mathrm{s}) = 5.78 \times 10^{8}\,\mathrm{Bq}.$$

A decay chain requires both production and loss terms: $\dot{N}_B = \lambda_A N_A - \lambda_B N_B$. Equal activities do not imply equal populations. Atomic-mass beta-plus Q values require subtracting $2m_e c^2$.







*Each half-life halves the activity; energy is released by moving towards the iron peak, by fusion from the left or fission from the right*

## Check conditions and vocabulary

Mixing atomic and nuclear masses without tracking electrons, or reporting a positive Q for a reaction that needs energy input. Balance charge and nucleon number first, then check the sign of Q.

**Q value** Q 值: The rest-mass energy converted to kinetic energy in a reaction, $(m_{initial} - m_{final})c^2$.

**activity**: The decay rate $\lambda N$ of a radioactive sample, measured in becquerel.



# Coordinate operators, separated modes and stable updates

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Writing the Laplacian of a radial function as f″ drops a term; the operator belongs to the coordinates, not the symbol.

Prerequisites: Algebra, differentiation and vector components.

- Use the divergence 散度 and Laplacian in the coordinate system selected by the symmetry
- Solve a simple separated boundary-value mode problem
- Check convergence, units and stability of an original numerical update

## Match operator to symmetry

Choose coordinates by symmetry before writing operators. For a radial function in spherical coordinates, $\nabla^2 f = f'' + (2/r)f'$, and the divergence of a radial field $A = A_r\,\hat{r}$ is $(1/r^2)d(r^2 A_r)/dr$. Check: $A = k\,\hat{r}/r^2$ has divergence $(1/r^2)dk/dr = 0$ away from the origin, and $f = 1/r$ satisfies $\nabla^2 f = f'' + 2f'/r = 2/r^3 - 2/r^3 = 0$ away from the origin. Cartesian forms apply only to Cartesian dependences.

## Separate the modes

Separation of variables 分离变量法 turns a boundary-value problem into ordinary modes. For a string fixed at both ends, write $u = X(x)T(t)$; the wave equation gives $X''/X = (1/c^2)T''/T = -k^2$, the fixed ends force $X = \sin(n\pi x/L)$, and the allowed frequencies are $\omega_n = n\pi c/L$, $f_n = nc/(2L)$. Each mode must satisfy the boundary conditions before any sum over modes.

## Bound the step

An explicit Euler update for $y' = -\lambda y$ is $y_{n+1} = (1 - \lambda h)y_n$, bounded when $|1 - \lambda h| \le 1$, i.e. $0 < h \le 2/\lambda$ for $\lambda > 0$; asymptotic decay needs $0 < h < 2/\lambda$. With $\lambda = 10$ /s, h must not exceed 0.2 s; h=0.25 s multiplies by −1.5 each step and diverges with alternating sign. Stability is a property of the recurrence, not of the true solution.

## Verify the answer

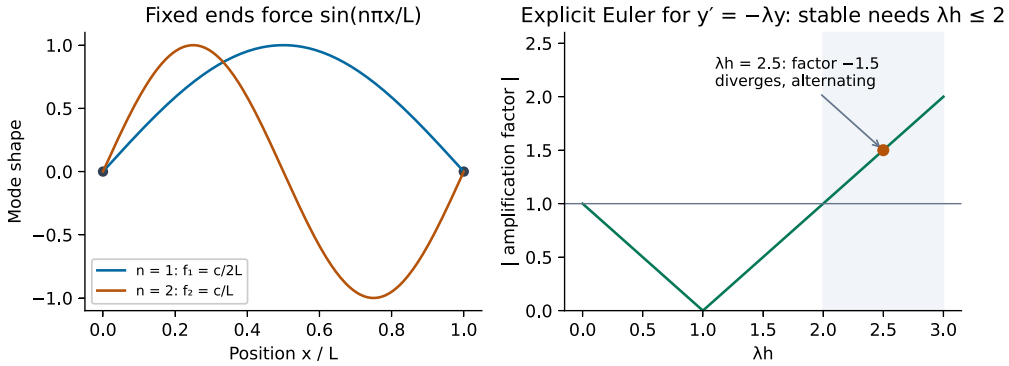
Check a numerical answer three ways before trusting it: units of every term ($\lambda h$ must be dimensionless), convergence under refinement (halve h and compare; explicit Euler error is first order), and a limiting case with a known exact answer. For $y' = -4y$ with y0=8 and h=0.2, five steps give $8 \times 0.2^5 = 0.00256$ versus exact $8e^{-4} \approx 0.1465$: stable but inaccurate, so refine.

## Worked method

A spherical radial field has divergence

$$\nabla \cdot (A_r \hat{r}) = r^{-2} \frac{d}{dr}(r^2 A_r).$$

For $A_r = Cr$, it gives 3C, not C. Fixed string ends require $X(0) = X(L) = 0$, hence $X_n = \sin(n\pi x/L)$ with positive integer n. Euler for $\dot{y} = -\lambda y$ gives $y_{n+1} = (1 - \lambda h) y_n$. For positive h, bounded absolute stability permits $h \leq 2/\lambda$; decay to zero requires the strict inequality $0 < h < 2/\lambda$. At the upper endpoint the numerical solution alternates without decay. Stability does not establish accuracy.




*Fixed ends force sin(nπx/L); explicit Euler for $y' = -\lambda y$ is stable only for $\lambda h \leq 2$*

## Check conditions and vocabulary

Applying the Cartesian Laplacian to a radial function, summing modes that violate the boundary conditions, or calling a bounded-looking run converged without refining the step. Match operator, boundaries and stability condition 稳定性条件 to the actual problem.

**separation of variables**: Solving a partial differential equation by writing the unknown as a product of single-variable factors.

**stability condition**: The step-size restriction that keeps a numerical recurrence from amplifying errors.