

Reaction energies, activity and decay balances

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Reaction energies, activity and decay balances

One atomic mass unit is 931.5 MeV, so a reaction energy is a small difference between large masses.

Prerequisites: 8, 25, 21.

- Compute reaction energy from complete atomic or nuclear mass bookkeeping
- Calculate activity from half-life and a simple decay-chain balance
- Compare fission and fusion binding changes with charge and nucleon conservation

Bookkeep the masses

The reaction energy is $Q=(m_{\text{initial}}-m_{\text{final}})c^2$; $Q>0$ releases kinetic energy. Track every species, and keep the mass convention consistent: tabulated atomic masses include electrons. For β^- decay and for α decay with a neutral helium product, atomic electron counts balance directly; β^+ or electron capture needs explicit electron-mass terms. Use $1\text{ u}=931.5\text{ MeV}/c^2$.

Compute Q and check balance

Alpha decay of ^{238}U with atomic masses $238.0508\text{ u} \rightarrow 234.0436\text{ u} + 4.0026\text{ u}$ gives $\Delta m=0.0046\text{ u}$ and $Q\approx 4.28\text{ MeV}$. Fusion $\text{D}+\text{T}$ with $2.0141+3.0161-4.0026-1.0087=0.0189\text{ u}$ gives $Q\approx 17.6\text{ MeV}$. A reaction also requires charge and nucleon-number balance: $^{14}\text{N}+\alpha\rightarrow^{17}\text{O}+\text{p}$ balances $7+2=9$ and $14+4=18$.

Convert half-life to activity

Activity is $A=\lambda N$ with decay constant $\lambda=\ln 2/T_{1/2}$; one becquerel is one decay per second. For $T_{1/2}=2\text{ h}$ and $N=6\times 10^{12}$, $A=6\times 10^{12}\times \ln 2/7200\approx 5.8\times 10^8\text{ Bq}$. After three half-lives both N and A fall by $1/8$. Equal activities do not imply equal atom counts: shorter half-life at fixed N gives larger A .

Compare binding changes

In a chain $A\rightarrow B\rightarrow C$ with a long-lived parent ($\lambda_A\ll\lambda_B$), B accumulates until its activity approaches the parent activity (secular equilibrium); write the balance $dN_B/dt=\lambda_A N_A-\lambda_B N_B$. The binding-energy-per-nucleon curve peaks near iron, so heavy fission (about 7.6 to 8.5 MeV per nucleon over roughly 240 nucleons, of order 200 MeV) and light fusion both release energy; compare total binding, not

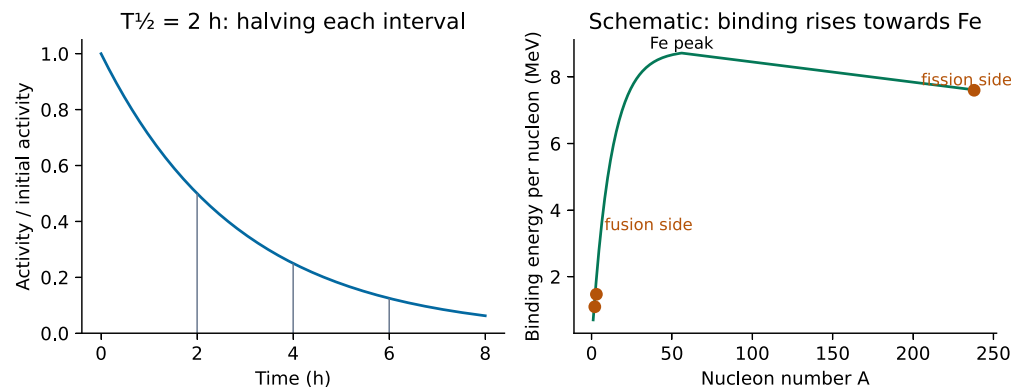
nucleon counts.

Worked method

Activity is $A=\lambda N$, with $\lambda=\ln 2/T_{1/2}$. For $N=6.0\times 10^{12}$ and half-life 2 hours, convert time before calculating.

$$A=(\ln 2)N/T_{1/2}=(\ln 2)(6.0\times 10^{12})/(7200\text{ s})=5.78\times 10^8\text{ Bq}.$$

A decay chain requires both production and loss terms: $\dot{N}_B=\lambda_A N_A-\lambda_B N_B$. Equal activities do not imply equal populations. Atomic-mass beta-plus Q values require subtracting $2m_e c^2$.



Reaction energies, activity and decay balances: original GRE teaching diagram.

Check conditions and vocabulary

Mixing atomic and nuclear masses without tracking electrons, or reporting a positive Q for a reaction that needs energy input. Balance charge and nucleon number first, then check the sign of Q .

Q value Q : The rest-mass energy converted to kinetic energy in a reaction, $(m_{\text{initial}}-m_{\text{final}})c^2$.

activity: The decay rate λN of a radioactive sample, measured in becquerel.