

Simultaneity, length measurement and velocity transformation

GRE · GRE Subject Test · GRE Physics

Two observers can disagree about which of two events happened first, and both can be right.

Prerequisites: 22.

- Transform event separations and identify proper length versus simultaneous endpoint readings
- Apply relativistic velocity addition and check a four-vector invariant
- Classify the interval sign to order events and rule out causal contact

Transform separations

Use the Lorentz transformation with declared frame motion: if frame S' moves at $+v$ along x relative to S , then $x'=\gamma(x-vt)$ and $t'=\gamma(t-vx/c^2)$ with $\gamma=1/\sqrt{1-v^2/c^2}$. Apply it to event separations Δx and Δt , not only to single coordinates. Two events simultaneous in S ($\Delta t=0$) at different places have $\Delta t'=-\gamma v \Delta x/c^2$: the event further in the $+x$ direction happens earlier in S' . Simultaneity is frame-dependent, not a bookkeeping error.

Measure length in one frame

A moving rod length must be read from endpoint positions at the same lab time ($\Delta t=0$). Then $\Delta x'=\gamma \Delta x$ gives $L=L_0/\gamma$ with L_0 the rest-frame (proper) length. Reading the endpoints simultaneously in the rod frame instead does not produce the lab length; the two reading conditions answer different questions. State the frame and the reading condition before quoting any length.

Classify the interval

The interval $s^2=c^2\Delta t^2-\Delta x^2$ (mostly-minus convention, declared once) is invariant. $s^2>0$ is timelike: a frame exists with the events co-located, and $\Delta\tau=s/c$ is the proper time between them. $s^2<0$ is spacelike: a frame exists with the events simultaneous, and $\sqrt{-s^2}$ is the proper distance. $s^2=0$ is lightlike. Timelike order is the same in every frame; spacelike order can swap, and no signal can join the events because $|\Delta x/\Delta t|>c$.

Add velocities and check invariants

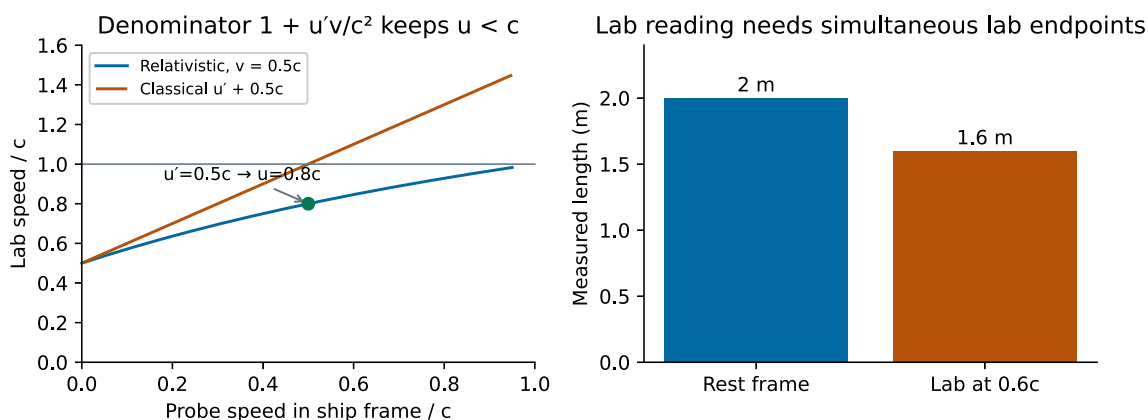
Collinear velocities add as $u=(u'+v)/(1+u'v/c^2)$; the denominator keeps $u < c$ for any $u', v < c$, and $u'=c$ gives $u=c$ exactly. Energy and momentum form a four-vector with invariant $E^2 - p^2 c^2 = m^2 c^4$: compute it in any convenient frame. Check each result in the nonrelativistic limit, where the addition rule reduces to $u=u'+v$ and the invariant reduces to the rest energy.

Worked method

Two lab events have $\Delta x = 900$ m and $\Delta t = 2$ microseconds. Their spacetime interval is spacelike because $c \Delta t = 600$ m is less than Δx . A frame making them simultaneous satisfies

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \quad v = c^2 \Delta t / \Delta x = 2c/3.$$

Their separation in that frame is $\sqrt{\Delta x^2 - c^2 \Delta t^2} = 670.8$ m. No frame can put these spacelike events at the same position.



Simultaneity, length measurement and velocity transformation: original GRE teaching diagram.

Check conditions and vocabulary

Quoting $L=L_0/\gamma$ without the simultaneous-endpoint condition, or adding velocities as $u=u'+v$ at relativistic speeds. A simultaneity difference is physical, not a mistake.

spacetime interval: The invariant combination $c^2 \Delta t^2 - \Delta x^2$ under the declared sign convention, whose sign classifies an event separation.

proper length: The endpoint separation measured in the object's rest frame; any other frame must read both endpoints simultaneously in its own time.