

Thermal spectra and one-electron scaling

GRE · GRE Subject Test · GRE Physics

A spectral peak, a total radiation flux and an atomic transition energy are three different observables.

Prerequisites: 7, 9, 34.

- Distinguish wavelength spectral peak, integrated radiation and photon energy
- Derive Bohr radius and energy scaling under one-electron assumptions
- Calculate Coulomb-ion transitions, series limits and ionisation thresholds

Identify the spectral measure

For ideal thermal equilibrium radiation, Planck's wavelength spectral radiance is $B_\lambda = 2hc^2 / \{\lambda^5 [\exp(hc/(\lambda k_B T)) - 1]\}$. B_λ is per wavelength interval and per solid angle, not a total power. A blackbody's hemispheric surface flux spectrum is $M_\lambda = \pi B_\lambda$; integrating over all wavelengths gives $\sigma_{SB} T^4$. All temperatures are absolute Kelvin. The wavelength peak obeys $\lambda_{max} T \approx 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$. A spectrum per frequency has a different peak because $B_\nu d\nu$ and $B_\lambda d\lambda$ include a Jacobian; c/λ_{max} is not the peak frequency of B_ν . Increasing T moves the wavelength peak shorter and increases the integrated flux by T^4 , not by the peak-position ratio alone.

Separate power from one photon

For an original uniform grey surface of area A and wavelength-independent emissivity ε , net radiative power to a large uniform environment is $\varepsilon \sigma_{SB} A (T^4 - T_{env}^4)$ under the stated view-factor assumptions. Use $\sigma_{SB} \approx 5.670 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$. Spectrally varying emissivity or incomplete surroundings requires a more detailed model. A photon at a specified wavelength has $E = hc/\lambda$, conveniently about $1240 \text{ eV} \cdot \text{nm}/\lambda_{nm}$. A thermal spectrum contains many photon energies; a photon at the wavelength peak is not the mean energy of every photon. For $T = 3000 \text{ K}$, $\lambda_{max} \approx 966 \text{ nm}$; its photon energy is about 1.284 eV . A surface with $A = 2 \times 10^{-4} \text{ m}^2$ and $\varepsilon = 0.5$ emits 459.27 W to a negligibly cold environment.

Derive one-electron scaling

In the historical Bohr one-electron model with a heavy nucleus of charge Ze , Coulomb force $m_e v^2/r = Ze^2/(4\pi\epsilon_0 r^2)$ and angular momentum $m_e vr = n\hbar$ lead to $r_n = a_0 n^2/Z$, $v_n = Z\alpha c/n$ and $E_n \approx -13.6 Z^2/n^2 \text{ eV}$. The radius scales with n^2/Z while binding energy scales with Z^2/n^2 ; do not use the same charge power in both. These circular-orbit assumptions are a historical scaling model. Wave-mechanical angular momentum is $\sqrt{l(l+1)}\hbar$ with $l=0, \dots, n-1$, so a $1s$ orbital has zero orbital angular momentum rather than the Bohr $n\hbar$ value. Reduced-mass corrections replace m_e with μ : Coulomb radius scales as $1/\mu$ and energy as μ . Multielectron screening,

large-Z relativistic corrections and fine structure are outside the simple model.

Subtract levels and locate limits

For an emission $n_i \rightarrow n_f$ with $n_i > n_f$, $E_\gamma \approx 13.6Z^2(1/n_f^2 - 1/n_i^2)$ eV. Convert this positive difference to $\lambda \approx 1240/E_\gamma$ nm. Ionisation from n requires energy $13.6Z^2/n^2$ eV to reach the continuum zero. Absorption reverses the level ordering and needs the corresponding positive incoming photon energy. For a series ending at fixed n_f , the largest bound-bound photon energy occurs as $n_i \rightarrow \infty$, giving $E_{\text{limit}} = 13.6Z^2/n_f^2$ and the shortest series wavelength. For one-electron helium $Z=2$, $n=3 \rightarrow 2$ gives 7.5556 eV and $\lambda \approx 164.12$ nm. Its Bohr $n=3$ radius is $4.5a_0$; this radius describes the historical orbit, not a universal radial mode for every l at $n=3$.

Worked method

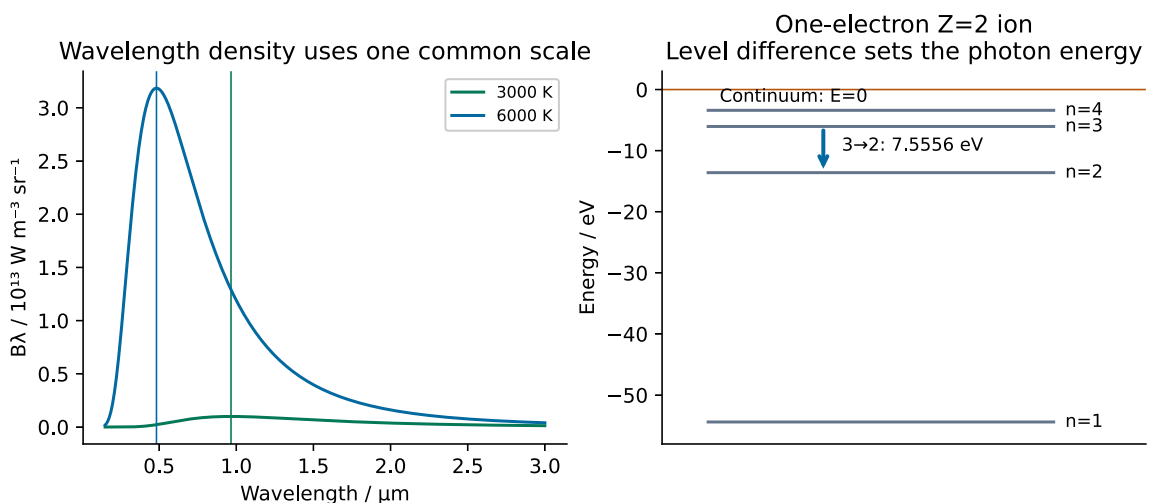
Wien's law refers to the wavelength-density peak of a blackbody spectrum.

$$\lambda_{\max} = b/T = (2.898 \times 10^{-3} \text{ m K})/(3000 \text{ K}) = 966 \text{ nm}.$$

A photon at that wavelength has energy

$$E_\gamma = hc/\lambda = (1240 \text{ eV nm})/(966 \text{ nm}) = 1.28 \text{ eV}.$$

The source emits a distribution of photon energies. Its integrated surface flux is σT^4 , and the frequency-density peak is not obtained by simply using c divided by this peak wavelength.



Thermal spectra and one-electron scaling: original GRE teaching diagram.

Check conditions and vocabulary

Declare a wavelength or frequency density, use Kelvin for fourth powers, and distinguish transitions from ionisation. A Bohr circular radius and angular momentum are not every wave orbital's radius and L .

spectral radiance: Radiation intensity per projected area, solid angle and stated spectral interval.

series limit : Limiting bound-bound photon energy or wavelength as the initial level approaches the continuum for a fixed final level.