

Signed lens and mirror images

GRE · GRE Subject Test · GRE Physics

A negative image distance is a geometric result that locates a virtual image, not a failed calculation to replace by its absolute value.

Prerequisites: 3.

- Locate paraxial thin-lens and spherical-mirror images with a declared sign convention
- Use signed magnification to classify orientation and real or virtual character
- Track successive image locations as the next element's object without resetting geometry

Declare distances and signs

Use the real-is-positive convention for this unit: a real object sends diverging incident rays into the element, so $s > 0$; an already converging incident beam can represent a virtual object, $s < 0$. A real image has $s' > 0$, while a virtual image has $s' < 0$. The paraxial equation is $1/f = 1/s + 1/s'$ and transverse magnification $M = -s'/s$. A converging lens has $f > 0$ and a diverging lens $f < 0$. For left-to-right light and a real object at $x < 0$ relative to a lens at $x = 0$, a real image lies at $x = s' > 0$; a negative s' puts its virtual image on the input side. State the convention before substitution, as other signed-coordinate conventions give different-looking equations.

Trace a lens image

For a converging thin lens, a ray parallel to the axis exits through the far focal point, and a ray through the ideal optical centre continues undeviated. Their intersection identifies a real image; backwards extensions can locate a virtual image. The paraxial approximation uses rays close to the axis and neglects thickness and aberrations. With $s > f > 0$, s' is positive and M is negative: the image is real and inverted. With $0 < s < f$, s' is negative and M is positive: the image is virtual, upright and enlarged. At $s = f$, emerging rays are parallel and no finite image plane exists. A diverging lens with a real object produces an upright reduced virtual image.

Apply reflected-image geometry

For a spherical mirror in the same real-is-positive convention, concave $f > 0$ and convex $f < 0$, with $f = R/2$ in the paraxial limit. A real reflected image is in front of the mirror on the incoming-light side; a virtual image is behind it. Thus a positive image distance has a different physical side for a mirror than for a transmitting lens. A parallel ray reflects through the concave focus; a ray aimed through the centre of curvature retraces its path. Convex reflected rays diverge as if from the focus behind the mirror. Use the same equation and $M = -s'/s$ with signed values, rather than silently combining mirror

geometry with a lens coordinate diagram.

Locate the next element's object

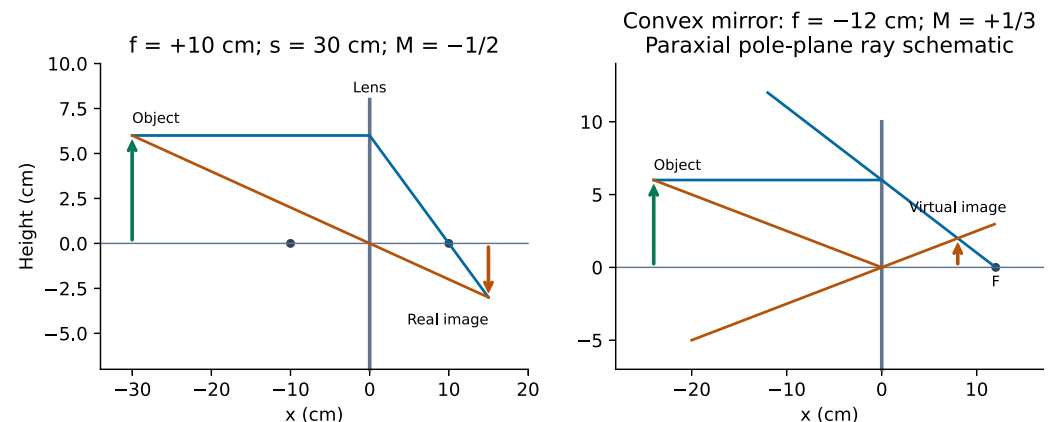
For a sequence of separated lenses, first calculate the actual image coordinate from the first lens. Relative to the next lens, decide whether the incoming rays diverge from a point before it or are still converging toward a point beyond it; these are real and virtual objects respectively. Only then assign its signed object distance and solve again. Total transverse magnification is the product of the individual signed magnifications for aligned paraxial elements. Element separation is not automatically the second object distance. Geometric real/virtual character depends on convergence, not on whether the final image is magnified. These ideal results do not model wave-optical resolution or spherical/chromatic aberration.

Worked method

Use real-is-positive distances and positive focal length for a converging lens. With $f = 12$ cm and real object distance $s = 8$ cm,

$$1/s' = 1/f - 1/s = 1/(12 \text{ cm}) - 1/(8 \text{ cm}) = -1/(24 \text{ cm}).$$

Thus $s' = -24$ cm and $M = -s'/s = +3$. The virtual image is upright, on the object's side. Signed distances identify the physical side; a negative distance is not an arithmetic failure.



Signed lens and mirror images: original GRE teaching diagram.

Check conditions and vocabulary

Keep signs until the geometry is interpreted; a positive mirror image lies on the incident side, while a positive transmitting-lens image lies on the outgoing side.

virtual image: An apparent image located by backwards ray extensions rather than actual outgoing-ray convergence.

transverse magnification: Signed image-height to object-height ratio, $-s'/s$ in the declared convention.