

Diffraction envelopes and missing interference orders

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Diffraction envelopes and missing interference orders

Two equal slits can have the right path difference for constructive interference and still deliver no intensity at that angle.

Prerequisites: 3, 17.

- Derive and evaluate the far-field single-slit intensity using amplitude superposition
- Separate double-slit interference spacing from finite-aperture envelope width
- Identify missing orders and state the limits of far-field and small-angle formulas

Add aperture amplitudes

For a uniformly illuminated slit of width a , far-field contributions across its opening arrive with a phase gradient $k \sin\theta$. Add their complex amplitudes before squaring: the normalised integral over x from $-a/2$ to $a/2$ is $\sin\beta/\beta$, where $\beta = \pi a \sin\theta/\lambda$. The intensity ratio is $(\sin\beta/\beta)^2$. At $\theta=0$ take the limit $\sin\beta/\beta \rightarrow 1$, rather than calling the centre undefined or dark. Zeros occur at $a \sin\theta = m\lambda$ with nonzero integer m . The central peak lies between the first zeros and is twice as wide as one adjacent zero-to-zero interval in $\sin\theta$. Side-peak maxima are not exactly halfway between their zeros.

Convert angles to the screen

At small angles on a distant screen L away, $y \approx L\theta$ and $\sin\theta \approx \theta$ give first zeros $y \approx \pm L\lambda/a$. Thus the central width is $2L\lambda/a$. Use metres consistently: a millimetre slit and a nanometre wavelength differ by six powers of ten. For a wider angle, use $\theta = a \sin(m\lambda/a)$ and $y = L \tan\theta$; the small-angle y expression then becomes inaccurate. Far-field conditions need nearly parallel rays from different parts of the aperture, for example L much greater than a^2/λ , or the equivalent focal-plane arrangement. Near-field Fresnel patterns cannot be assigned this intensity law blindly.

Multiply interference and envelope

For two coherent identical uniformly illuminated slits of width a and centre separation d , the normalised pattern is $(\sin\beta/\beta)^2 \cos^2\alpha$, with $\alpha = \pi d \sin\theta/\lambda$ and central intensity as the normalisation. The $\cos^2$ factor produces interference orders $d \sin\theta = n\lambda$; the single-slit factor gives the broader envelope zeros. Small-angle neighbouring interference spacing is $\lambda L/d$. Increasing d narrows fringe spacing; increasing a narrows the envelope. These are separate changes. The equal-height narrow-slit interference

formula alone does not predict the diminishing brightness or missing orders of finite apertures. Incoherent sources do not maintain the same phase-dependent cross term.

Cancel coincident orders

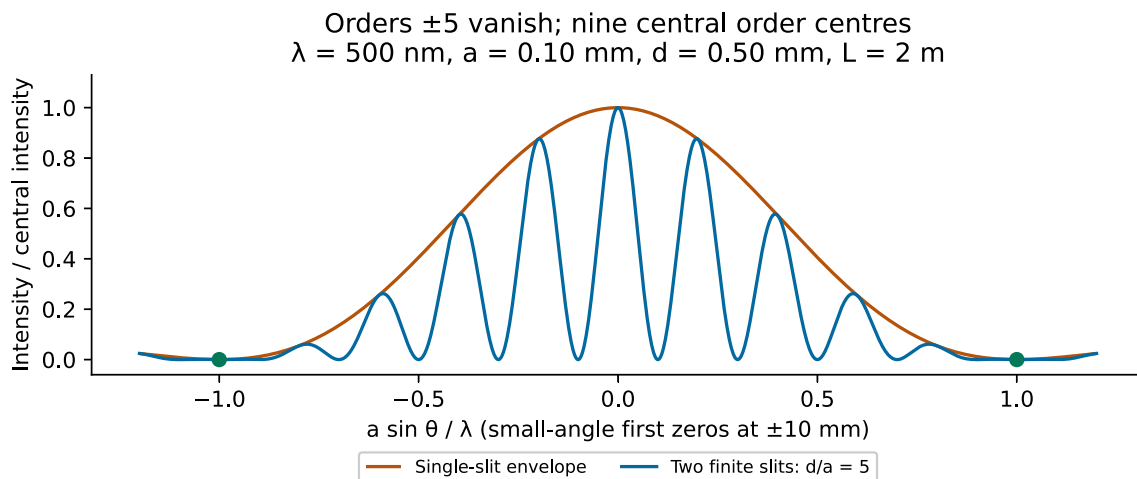
A missing order occurs when $n\lambda/d = m\lambda/a$ simultaneously, giving $n = m d/a$. If d/a is an integer r , orders $\pm r, \pm 2r$ and so on are cancelled by aperture zeros. Do not count an order at an envelope boundary as a visible bright fringe. In the central envelope, the nominal interference-order centres satisfy $|n| < d/a$; when r is an integer there are $2r-1$ such centres. For finite slit width, exact local maxima are shifted slightly by the changing envelope, so the interference-order locations are an approximation to observed peak centres. Identify what quantity is being requested before treating every $\cos^2$ maximum as an exact maximum of the product.

Worked method

For two finite slits, separate interference spacing from the single-slit envelope.

$$I/I_{\text{centre}} = (\sin \beta / \beta)^2 \cos^2 \alpha, \quad \beta = \pi a \sin \theta / \lambda, \quad \alpha = \pi d \sin \theta / \lambda.$$

If $d = 5a$, the first envelope zero $a \sin \theta = \lambda$ coincides with order $n = 5$. Nominal order centres in the central envelope are $n = -4$ through $+4$: nine in total. These are interference centres; the envelope can shift the exact product maxima slightly.



Diffraction envelopes and missing interference orders: original GRE teaching diagram.

Check conditions and vocabulary

Add field amplitudes before squaring, distinguish a from d , and exclude dark boundary orders. An interference maximum alone does not guarantee a maximum of the full pattern.

diffraction envelope : The aperture-dependent intensity factor that modulates the interference pattern.

missing order: An interference order cancelled because it coincides with an aperture intensity zero.