

Fluid continuity, pressure energy and viscous flow

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Fluid continuity, pressure energy and viscous flow

A narrow pipe carries the same steady flow rate as a wide pipe, but its pressure behaviour depends on both speed and viscous losses.

Prerequisites: 10, 47.

- Combine incompressible continuity with pressure, height and kinetic energy
- Apply hydrostatic and Bernoulli relations only under their stated conditions
- Use viscosity, Poiseuille scaling and Reynolds number to distinguish flow regimes

Conserve the flow rate

For steady flow, the mass passing successive cross-sections per unit time is equal: ρAv is constant. An incompressible liquid has nearly constant ρ , so $Q=Av$ is the volume flow rate. Halving pipe radius quarters area and multiplies mean speed by four for the same Q . Pressure does not determine Q without a model of the rest of the system. In a stationary liquid, $dp/dz=-\rho g$ with z upward; therefore pressure increases by ρgh a distance h below a surface. Use absolute or gauge pressure consistently on both sides.

Account for pressure energy

For steady, incompressible, inviscid motion along a streamline with no pump or dissipative loss, Bernoulli gives $p+\rho v^2/2+\rho gz=\text{constant}$. These three terms are energy per volume and have pressure units. At equal height, larger speed requires smaller static pressure under these assumptions. A higher outlet also uses pressure/kinetic energy to gain gravitational energy. Bernoulli along one streamline need not imply the same constant on different streamlines in rotational flow. A stagnation point has $v=0$ and converts local speed energy to a pressure rise in the ideal model.

Include viscous loss

Viscosity transports momentum between neighbouring fluid layers. For steady fully developed laminar flow of a Newtonian incompressible liquid in a circular tube, $Q=\pi R^4 \Delta p/(8\eta L)$. Here Δp is the pressure drop along the tube, η dynamic viscosity and L tube length. Doubling radius at fixed Δp , η and L multiplies Q by sixteen. Doubling Q at fixed geometry needs twice the pressure drop. This viscous pressure loss cannot be added to a loss-free Bernoulli equation as though nothing changes; include a dissipative pressure/head loss or use the viscous model.

Check the flow regime

The Reynolds number $Re = \rho v D / \eta$ compares inertial and viscous effects using characteristic speed v and length D . For a pipe use mean speed and internal diameter, not radius. Re is dimensionless: density times speed times length has the same units as dynamic viscosity. Small Re favours viscous dominance; transition to turbulence depends on geometry and disturbances, so a single threshold is not a universal law. Poiseuille scaling is not safe after assuming a turbulent flow. Check volume continuity, sign of pressure change and the regime before selecting a formula.

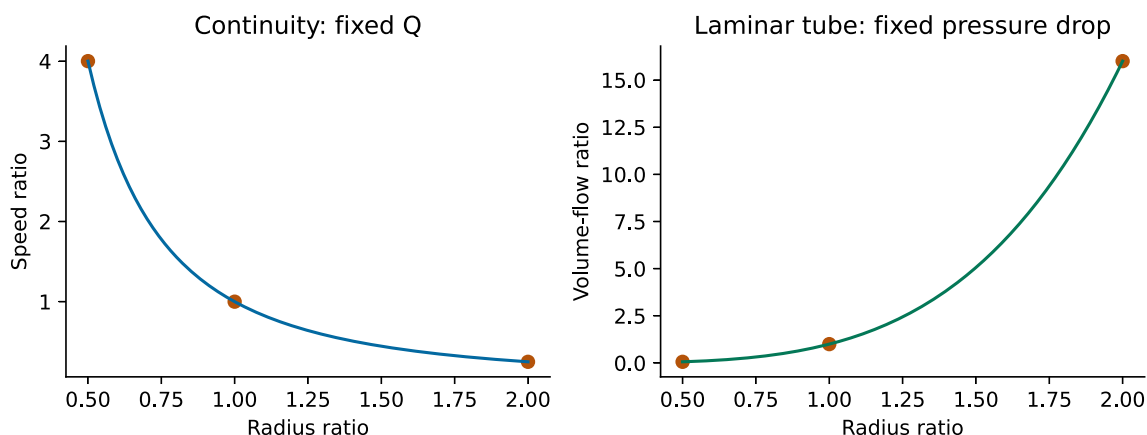
Worked method

A steady incompressible horizontal ideal flow has areas 4 and 1 square centimetres. At the first section speed is 1 m/s. Continuity and Bernoulli give

$$v_2 = A_1 v_1 / A_2 = (4 \text{ cm}^2)(1 \text{ m/s}) / (1 \text{ cm}^2) = 4 \text{ m/s}.$$

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) = \frac{1}{2} (1000 \text{ kg/m}^3) [(4 \text{ m/s})^2 - (1 \text{ m/s})^2] = 7500 \text{ Pa}.$$

These relations assume no pump, viscosity loss or height change. A real narrow tube may violate them.



Fluid continuity, pressure energy and viscous flow: original GRE teaching diagram.

Check conditions and vocabulary

Continuity keeps Q constant across one steady pipe; Poiseuille compares Q between different systems at a specified pressure drop. State what is held fixed before comparing radius powers.

volume flow rate: Volume crossing a section per unit time, $Q = Av$ for mean speed v .

dynamic viscosity: The coefficient relating shear stress to velocity gradient in a Newtonian fluid.