

Accelerating and rotating reference frames

GRE · GRE Subject Test · GRE Physics

A person standing still on a turning platform needs a real inward force, although their platform-relative acceleration is zero.

Prerequisites: 26.

- Transform acceleration with translating and rotating frame $\vec{a} = \vec{a}_{\text{real}} - \vec{a}_{\text{frame}}$ terms
- Determine centrifugal, Coriolis and Euler directions from cross products
- Separate apparent forces from real interactions and test inertial limits

Translate the frame

In a translating frame with origin acceleration $\vec{A}$, Newton's equation becomes $\vec{a}' = \vec{F}_{\text{real}} - m\vec{A}$. An upward-accelerating elevator therefore has $N - mg = ma$ in the ground frame, or $N - mg - ma = 0$ for its stationary passenger in the elevator frame. These are the same prediction. The additional term is an apparent force caused by the chosen accelerating coordinates; it is not a new contact with another body. Uniform translation with $\vec{A} = 0$ changes velocity but needs no apparent force.

Differentiate rotating axes

For a rotating basis, differentiating a vector adds $\vec{\Omega} \times$ that vector. Applying this twice gives $\vec{a} = \vec{a}_{\text{origin}} + \vec{a}' + 2\vec{\Omega} \times \vec{v}' + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + \dot{\vec{\Omega}} \times \vec{r}$. Here $\vec{r}$ and $\vec{v}'$ are measured relative to the moving origin in its rotating axes; all vectors in a calculation must be expressed in the same basis at the same instant. Move the last three rotational terms to the force side to obtain Coriolis $-2m\vec{\Omega} \times \vec{v}'$, centrifugal $-m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$, and Euler $-m\dot{\vec{\Omega}} \times \vec{r}$. A constant rotation removes the Euler term, not the Coriolis term.

Check cross-product directions

Take $\vec{\Omega}$ along $+z$, an anticlockwise platform viewed from above, and a particle at $\vec{r}$ along $+x$. Centrifugal force $-m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ is along $+x$ with magnitude $m\Omega^2 r$. If the particle moves outward with $\vec{v}'$ along $+x$, then $\vec{\Omega} \times \vec{v}'$ is $+y$, so Coriolis force $-2m\vec{\Omega} \times \vec{v}'$ points $-y$. Reversing the relative velocity reverses Coriolis; keeping $\vec{v}' = 0$ makes it vanish. Coriolis is perpendicular to $\vec{v}'$ and does no instantaneous work on that relative motion. Centrifugal force is outward from the rotation axis, not necessarily outward from the chosen origin in an arbitrary three-dimensional position.

Balance real and apparent forces

A body at rest on the platform has $\vec{a}' = \vec{v}' = 0$. With constant rotation and a fixed origin, its real force must be $m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$, inward, cancelling the outward apparent term in the rotating equation. If rotation changes, a tangential real force must also balance the

Euler term. Always check $\Omega \rightarrow 0$ and $A \rightarrow 0$: ordinary inertial Newtonian motion must return. Do not mix the real inward centripetal requirement with an added outward real reaction on the same body; interaction partners belong in separate free-body diagrams.

Worked method

In a rotating frame, Coriolis force uses the relative velocity.

$$\mathbf{F}_C = -2m\boldsymbol{\Omega} \times \mathbf{v}', \quad \mathbf{F}_{cf} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

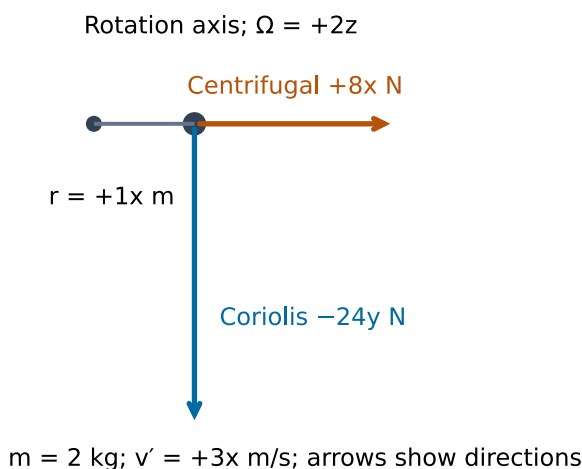
For $m = 2 \text{ kg}$, $\Omega = 2 \text{ rad/s}$ along z , $r = 1 \text{ m}$ along x and $v' = 3 \text{ m/s}$ along x ,

$$F_{cf} = m\Omega^2 r = (2 \text{ kg})(2 \text{ rad/s})^2(1 \text{ m}) = 8 \text{ N} \quad (+x).$$

$$F_C = 2m\Omega v' = 2(2 \text{ kg})(2 \text{ rad/s})(3 \text{ m/s}) = 24 \text{ N} \quad (-y).$$

Euler force is zero only because Ω is constant. These are coordinate terms, not extra physical contacts.

Constant anticlockwise frame; apparent forces



Accelerating and rotating reference frames: original GRE teaching diagram.

Check conditions and vocabulary

The relative velocity v' belongs in the Coriolis term. Using the full inertial velocity counts rotation twice.

Coriolis force: The apparent rotating-frame force $-2m\boldsymbol{\Omega} \times \mathbf{v}'$ caused by relative motion.

centrifugal force: The apparent rotating-frame force $-m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$, directed away from the rotation axis.